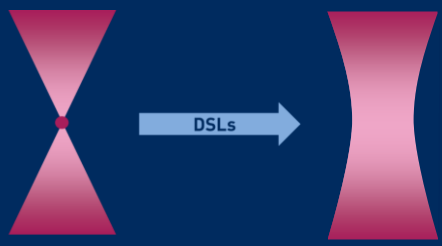

Program Generation for Performance
Spiral

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Computer Science
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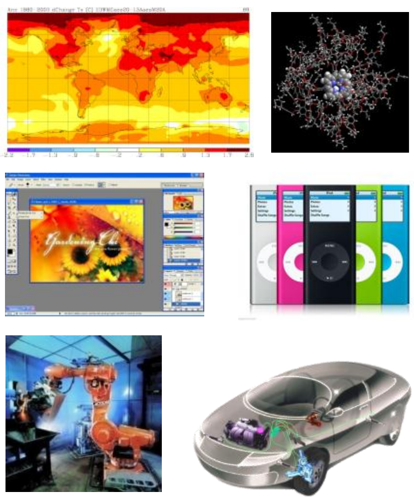
Mathematical Computing



Science simulations

Audio, image, Video processing

Signal processing, communication, control

Security

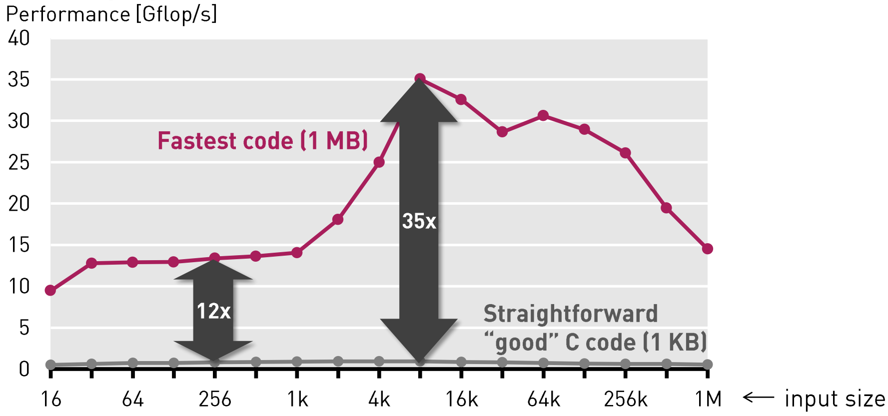
Machine learning, data analytics

Optimization

Highest performance is often crucial

Example: Discrete Fourier Transform

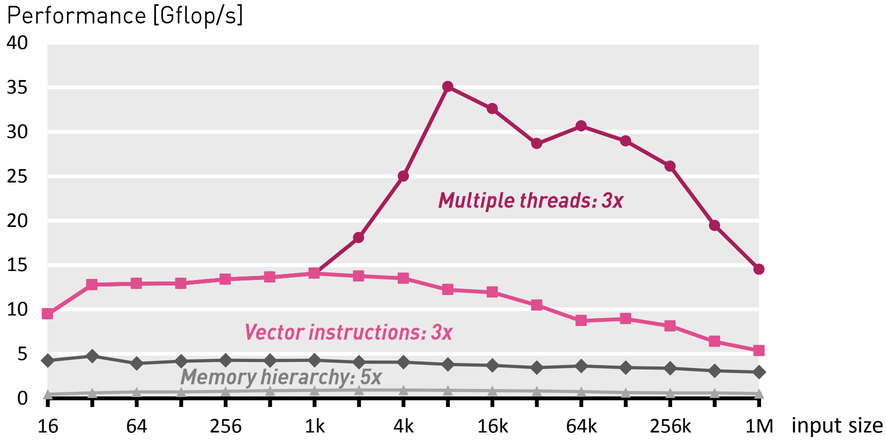
DFT (single precision) on Intel Core i7 (4 cores, 2.66 GHz)



Vendor compiler, best flags

Roughly same operations count

DFT (single precision) on Intel Core i7 (4 cores, 2.66 GHz)



Compiler doesn't do the job

Doing by hand = restructure algorithm for locality & parallelism, handle choices, choose proper code style, use vector intrinsics, ...
= nightmare

Model predictive control
Eigenvalues
LU factorization
Optimal binary search organization
Image color conversions
Image geometry transformations
Enclosing ball of points
Metropolis algorithm, Monte Carlo
Seam carving
SURF feature detection
Submodular function optimization
Graph cuts, Edmond-Karps Algorithm
Gaussian filter
Black Scholes option pricing
Disparity map refinement

Singular-value decomposition
Mean shift algorithm for segmentation
Stencil computations
Displacement based algorithms
Motion estimation
Multiresolution classifier
Kalman filter
Object detection
IIR filters
Arithmetic for large numbers
Optimal binary search organization
Software defined radio
Shortest path problem
Feature set for biomedical imaging
Biometrics identification

Same for most computational problems:
Straightforward code is highly suboptimal

Optimization: Register Locality and ILP

```
// straightforward code
for(i = 0; i < N; i += 1)
  for(j = 0; j < N; j += 1)
    for(k = 0; k < N; k += 1)
      c[i][j] += a[i][k]*b[k][j];
```

Concise and slow

Removes aliasing

Enables register allocation and instruction scheduling

Compiler does not do well:

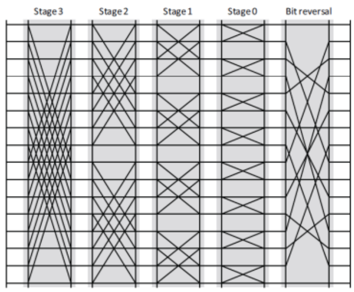
- often illegal
- many choices

```
// unrolling + scalar replacement
for(i = 0; i < N; i += MU) {
    for(j = 0; j < N; j += NU) {
      for(k = 0; k < N; k += KU) {
        t1 = A[i*N + k];
        t2 = A[i*N + k + 1];
        t3 = A[i*N + k + 2];
        t4 = A[i*N + k + 3];
        t5 = A[(i + 1)*N + k];
        <more copies>

        t10 = t1  * t9;
        t17 = t17 + t10;
        t21 = t1  * t8;
        t18 = t18 + t21;
        t12 = t5  * t9;
        t19 = t19 + t12;
        t13 = t5  * t8;
        t20 = t20 + t13;
        <more ops>

        C[i*N + j]         = t17;
        C[i*N + j + 1]     = t18;
        C[(i+1)*N + j]     = t19;
        C[(i+1)*N + j + 1] = t20;
      }
    }
  }
```

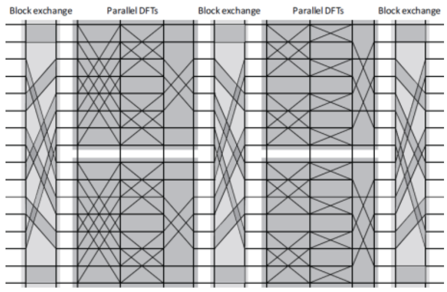
load
compute
store

Ugly and fast

Optimization for Parallelism (Threads)



Parallelism is present, but is not in the "right shape"



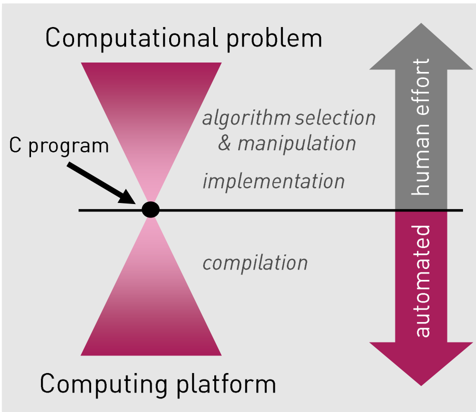
Restructured for locality and parallelism (shared memory, 2 cores, 2 elements per cache line)

Compiler usually does not do

- analysis may be unfeasible
- may require algorithm changes
- may require domain knowledge
- may require processor parameters

Current practice: **Thousands of programmers re-implement and re-optimize the same functionality for every new processor and for every new processor generation**

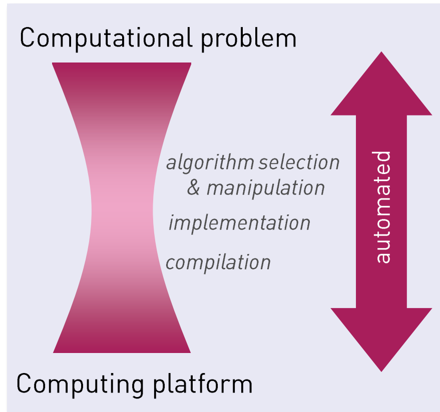
Current



C code is a singularity:

- Compiler has no access to high level information
- No structural optimization
- No evaluation of choices

Future

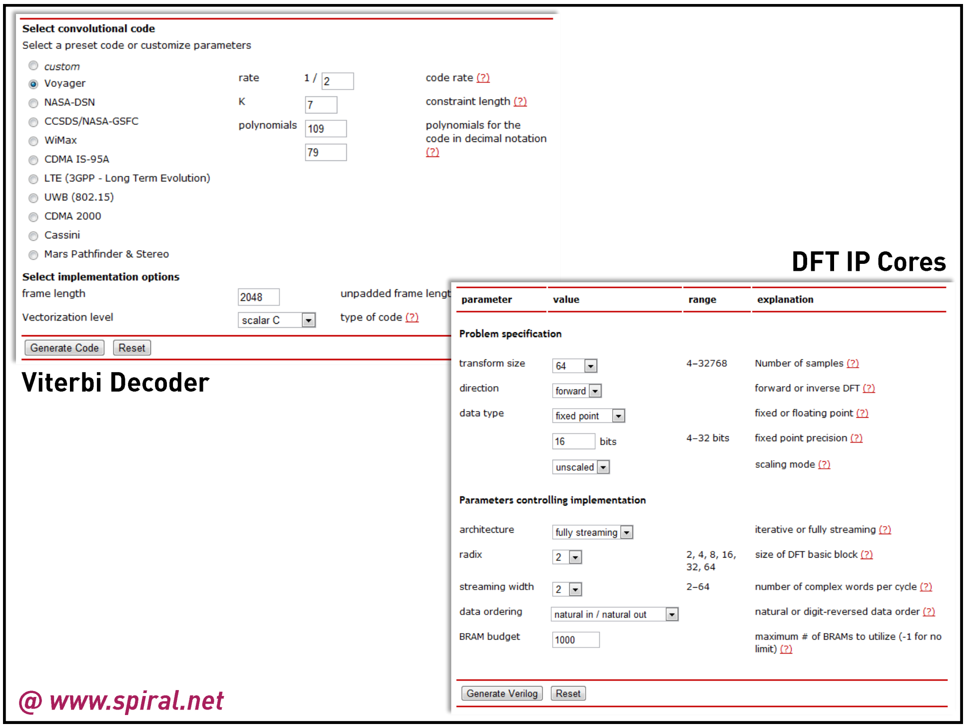


Challenge: conquer the high abstraction level for ***more/complete automation***

DSLs!

Goal:

Computer generation of high performance code for ubiquitous performance-critical components



Possible Approach:

Capturing algorithm knowledge:
Domain-specific languages (DSLs)

Structural optimization:
Rewriting systems

High performance code style:
Compiler

Decision making for choices:
Machine learning

Spiral: Program Generation for Performance (www.spiral.net)

Linear Transforms

$$\begin{pmatrix} y_0 \\ y_1 \\ \vdots \\ y_{n-1} \end{pmatrix} = y = Tx \longleftarrow \boxed{T\cdot} \longleftarrow x = \begin{pmatrix} x_0 \\ x_1 \\ \vdots \\ x_{n-1} \end{pmatrix}$$

Output ***Input***

Example: $T = \mathbf{DFT}_n = [e^{-2k\ell\pi i/n}]_{0\le k,\ell<n}$

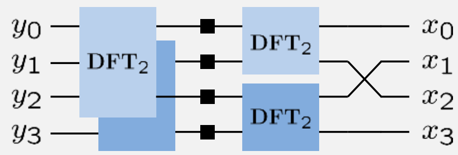
Algorithms: Example FFT, n = 4

Fast Fourier transform (FFT):

$$\begin{bmatrix}1&1&1&1\\1&i&-1&-i\\1&-1&1&-1\\1&-i&-1&i\end{bmatrix}x = \begin{bmatrix}1&\cdot&1&\cdot\\\cdot&1&\cdot&1\\1&\cdot&-1&\cdot\\\cdot&1&\cdot&-1\end{bmatrix}\begin{bmatrix}1&\cdot&\cdot&\cdot\\\cdot&1&\cdot&\cdot\\\cdot&\cdot&1&\cdot\\\cdot&\cdot&\cdot&i\end{bmatrix}\begin{bmatrix}1&1&\cdot&\cdot\\1&-1&\cdot&\cdot\\\cdot&\cdot&1&1\\\cdot&\cdot&1&-1\end{bmatrix}\begin{bmatrix}1&\cdot&\cdot&\cdot\\\cdot&\cdot&1&\cdot\\\cdot&1&\cdot&\cdot\\\cdot&\cdot&\cdot&1\end{bmatrix}x$$

12 adds, 4 mults | **4 adds** | **1 mult** | **4 adds** | **0 adds/mults**

Data flow graph



Description with matrix algebra (SPL)

$$\mathbf{DFT}_4 = (\mathbf{DFT}_2 \otimes \mathrm{I}_2)\,\mathrm{T}^4_2(\mathrm{I}_2 \otimes \mathbf{DFT}_2)\,\mathrm{L}^4_2$$

Decomposition Rules (>200 for >40 Transforms)

$$\mathbf{DFT}_n \to P^\top_{k/2,2m}\left(\mathbf{DFT}_{2m} \oplus \left(I_{k/2-1} \otimes_i C_{2m}\,\mathbf{rDFT}_{2m}(i/k)\right)\right)\left(\mathbf{RDFT}'_k \otimes I_m\right), \quad k \text{ even},$$

$$\begin{vmatrix}\mathbf{RDFT}_n\\\mathbf{RDFT}'_n\\\mathbf{DHT}_n\\\mathbf{DHT}'_n\end{vmatrix} \to (P^\top_{k/2,m} \otimes I_2)\left(\begin{vmatrix}\mathbf{RDFT}_{2m}\\\mathbf{RDFT}'_{2m}\\\mathbf{DHT}_{2m}\\\mathbf{DHT}'_{2m}\end{vmatrix} \oplus \left(I_{k/2-1} \otimes_i D_{2m}\begin{vmatrix}\mathbf{rDFT}_{2m}(i/k)\\\mathbf{rDFT}_{2m}(i/k)\\\mathbf{rDHT}_{2m}(i/k)\\\mathbf{rDHT}_{2m}(i/k)\end{vmatrix}\right)\right)\left(\begin{vmatrix}\mathbf{RDFT}'_k\\\mathbf{RDFT}'_k\\\mathbf{DHT}'_k\\\mathbf{DHT}'_k\end{vmatrix} \otimes I_m\right), \quad k \text{ even},$$

$$\begin{vmatrix}\mathbf{rDFT}_{2n}(u)\\\mathbf{rDHT}_{2n}(u)\end{vmatrix} \to L^{2n}_m\left(I_k \otimes_i \begin{vmatrix}\mathbf{rDFT}_{2m}((i+u)/k)\\\mathbf{rDHT}_{2m}((i+u)/k)\end{vmatrix}\right)\left(\begin{vmatrix}\mathbf{rDFT}_{2k}(u)\\\mathbf{rDHT}_{2k}(u)\end{vmatrix} \otimes I_m\right),$$

$$\mathbf{RDFT\text{-}3}_n \to (Q^\top_{k/2,m} \otimes I_2)\,(I_k \otimes_i \mathbf{rDFT}_{2m})(i+1/2)/k))\,(\mathbf{RDFT\text{-}3}_k \otimes I_m), \quad k \text{ even},$$

$$\mathbf{DCT\text{-}2}_n \to P^\top_{k/2,2m}\left(\mathbf{DCT\text{-}2}_{2m}K^{2m}_2 \oplus \left(I_{k/2-1} \otimes N_{2m}\,\mathbf{RDFT\text{-}3}^\top_{2m}\right)\right)B_n(L^{n/2}_{k/2} \otimes I_2)(I_m \otimes \mathbf{RDFT}'_k)Q_{m/2,k},$$

$\mathbf{DCT\text{-}3}_n \to$ [illegible]

Rules = algorithm knowledge

(≈100 journal papers)

$$\mathbf{DCT\text{-}4}_n \to Q^\top_{k/2,2m}\left(I_{k/2} \otimes N_{2m}\,\mathbf{RDFT\text{-}3}^\top_{2m}\right)B'_n(L^{n/2}_{k/2} \otimes I_2)(I_m \otimes \mathbf{RDFT\text{-}3}_k)Q_{m/2,k}.$$

$\mathbf{DFT}_n \to$ [illegible] $, \quad n = km$

$$\mathbf{DFT}_n \to P_n(\mathbf{DFT}_k \otimes \mathbf{DFT}_m)Q_n, \quad n = km,\ \gcd(k,m) = 1$$

$$\mathbf{DFT}_p \to R^T_p(\mathrm{I}_1 \oplus \mathbf{DFT}_{p-1})D_p(\mathrm{I}_1 \oplus \mathbf{DFT}_{p-1})R_p, \quad p \text{ prime}$$

$$\mathbf{DCT\text{-}3}_n \to (\mathrm{I}_m \oplus \mathrm{J}_m)\,\mathrm{L}^n_m(\mathbf{DCT\text{-}3}_m(1/4) \oplus \mathbf{DCT\text{-}3}_m(3/4)) \cdot(\mathrm{F}_2 \otimes \mathrm{I}_m)\begin{bmatrix}\mathrm{I}_m & 0 \oplus -\mathrm{J}_{m-1}\\ & \frac{1}{\sqrt{2}}(\mathrm{I}_1 \oplus 2\,\mathrm{I}_m)\end{bmatrix}, \quad n = 2m$$

$$\mathbf{DCT\text{-}4}_n \to S_n\mathbf{DCT\text{-}2}_n\,\mathrm{diag}_{0\le k<n}(1/(2\cos((2k+1)\pi/4n)))$$

$$\mathbf{IMDCT}_{2m} \to (\mathrm{J}_m \oplus \mathrm{I}_m \oplus \mathrm{I}_m \oplus \mathrm{J}_m)\left(\left(\begin{bmatrix}1\\-1\end{bmatrix} \otimes \mathrm{I}_m\right) \oplus \left(\begin{bmatrix}-1\\-1\end{bmatrix} \otimes \mathrm{I}_m\right)\right)\mathrm{J}_{2m}\,\mathbf{DCT\text{-}4}_{2m}$$

$$\mathbf{WHT}_{2^k} \to \prod_{i=1}^{t}(\mathrm{I}_{2^{k_1+\cdots+k_{i-1}}} \otimes \mathbf{WHT}_{2^{k_i}} \otimes \mathrm{I}_{2^{k_{i+1}+\cdots+k_t}}), \quad k = k_1 + \cdots + k_t$$

$$\mathbf{DFT}_2 \to \mathrm{F}_2$$

$$\mathbf{DCT\text{-}2}_2 \to \mathrm{diag}(1, 1/\sqrt{2})\,\mathrm{F}_2$$

$$\mathbf{DCT\text{-}4}_2 \to \mathrm{J}_2\,\mathrm{R}_{13\pi/8}$$

SPL to Code

SPL S	Pseudo code for $y = Sx$
$A_n B_n$	<code for: t = Bx> <code for: y = At>
$I_m \otimes A_n$	for (i=0; i<m; i++) <code for: y[i*n:1:i*n+n-1] = A(x[i*n:1:i*n+n-1])>
$A_m \otimes I_n$	for (i=0; i<n; i++) <code for: y[i:n:i+m*n-n] = A(x[i:n:i+m*n-n])>
D_n	for (i=0; i<n; i++) y[i] = D[i]*x[i];

L_k^{km}	for (i=0; i<k; i++)
	for (j=0; j<m; j++)
	y[i*m+j] = x[j*k+i];

F_2	y[0] = x[0] + x[1]; y[1] = x[0] - x[1];

$$\mathrm{I}_m \otimes A_n = \begin{bmatrix} A_n & & \\ & \ddots & \\ & & A_n \end{bmatrix}$$

Gives reasonable, straightforward code

Program Generation in Spiral

Transform DFT_8

Decomposition rules (algorithm knowledge)

Algorithm *(SPL)* $(\mathrm{DFT}_2 \otimes I_4)\, T_4^8\, (I_2 \otimes ((\mathrm{DFT}_2 \otimes I_2)\, T_2^4\, (I_2 \otimes \mathrm{DFT}_2)\, L_2^4))\, L_2^8$

parallelization
vectorization

Algorithm *(-SPL)* $\sum (S_j\, \mathrm{DFT}_2\, G_j) \sum (\sum (S_{k,l}\, \mathrm{diag}(\mathrm{t}_{k,l})\, \mathrm{DFT}_2\, G_l)\, \sum (S_m\, \mathrm{diag}(\mathrm{t}_m)\, \mathrm{DFT}_2\, G_{k,m}))$

locality
optimization

C Program

```
void sub(double *y, double *x) {
double f0, f1, f2, f3, f4, f7, f8, f10, f11;
    f0 = x[0] - x[3];
    f1 = x[0] + x[3];
    f2 = x[1] - x[2];
    f3 = x[1] + x[2];
    f4 = f1 - f3;
    y[0] = f1 + f3;
    y[2] = 0.7071067811865476 * f4;
    f7 = 0.9238795325112867 * f0;
< more lines >
```

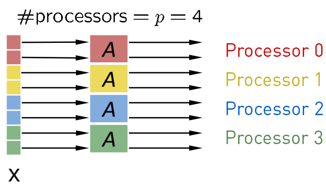
code style
code level
optimization

+ Search or Learning for Choices

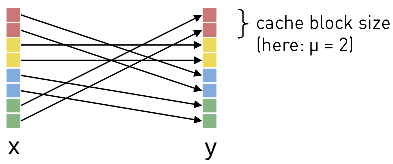
SPL to Shared Memory Code: Basic Idea

"Good" SPL structures

$$y = \left(\mathrm{I}_p \otimes A\right)x$$



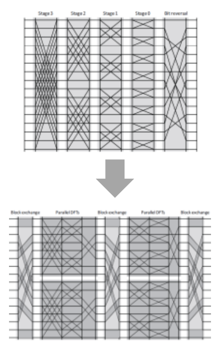
$$y = \left(P \otimes \mathrm{I}_\mu\right)x$$



Rewriting: Bad structures ➡ good structures

Example: SMP Parallelization

Franchetti, Voronenko & P, SC 2006



$$\underbrace{\mathbf{DFT}_{mn}}_{\mathsf{smp}(p,\mu)} \to \underbrace{\left((\mathbf{DFT}_m \otimes \mathrm{I}_n)\mathrm{T}^{mn}_n(\mathrm{I}_m \otimes \mathbf{DFT}_n)\,\mathrm{L}^{mn}_m\right)}_{\mathsf{smp}(p,\mu)}$$

...

$$\to \underbrace{\left(\mathbf{DFT}_m \otimes \mathrm{I}_n\right)}_{\mathsf{smp}(p,\mu)} \underbrace{\mathrm{T}^{mn}_n}_{\mathsf{smp}(p,\mu)} \underbrace{\left(\mathrm{I}_m \otimes \mathbf{DFT}_n\right)}_{\mathsf{smp}(p,\mu)} \underbrace{\mathrm{L}^{nm}_m}_{\mathsf{smp}(p,\mu)}$$

...

$$\to \left((\mathrm{L}^{mp}_m \otimes \mathrm{I}_{n/p\mu}) \otimes \mathrm{I}_\mu\right)\left(\mathrm{I}_p \otimes (\mathbf{DFT}_m \otimes \mathrm{I}_{n/p})\right)\left((\mathrm{L}^{mp}_p \otimes \mathrm{I}_{n/p\mu}) \otimes \mathrm{I}_\mu\right)$$
$$\left(\bigoplus_{i=0}^{p-1}{}_{||}\, \mathrm{T}^{mn,i}_n\right)\left(\mathrm{I}_p \otimes (\mathrm{I}_{m/p} \otimes \mathbf{DFT}_n)\right)\left(\mathrm{I}_p \otimes \mathrm{L}^{mn/p}_{m/p}\right)\left((\mathrm{L}^{pn}_p \otimes \mathrm{I}_{m/p\mu}) \otimes \mathrm{I}_\mu\right)$$

load-balanced, no false sharing

One rewriting system for every platform paradigm:
SIMD, distributed memory parallelism, FPGA, ...

Same Approach for Different Paradigms

Threading:

$$\underbrace{\mathbf{DFT}_{mn}}_{\text{smp}(p,\mu)} \to \underbrace{\left((\mathbf{DFT}_m \otimes \mathrm{I}_n)\mathsf{T}^{mn}_n(\mathrm{I}_m \otimes \mathbf{DFT}_n)\,\mathsf{L}^{mn}_m\right)}_{\text{smp}(p,\mu)}$$

$$\dots$$

$$\to \underbrace{\left(\mathbf{DFT}_m \otimes \mathrm{I}_n\right)}_{\text{smp}(p,\mu)} \underbrace{\mathsf{T}^{mn}_n}_{\text{smp}(p,\mu)} \underbrace{\left(\mathrm{I}_m \otimes \mathbf{DFT}_n\right)}_{\text{smp}(p,\mu)} \underbrace{\mathsf{L}^{nm}_m}_{\text{smp}(p,\mu)}$$

$$\dots$$

$$\to \left((\mathsf{L}^{mp}_m \otimes \mathrm{I}_{n/p\mu}) \otimes_\mu \mathrm{I}_\mu\right)\left(\mathrm{I}_p \otimes_\| (\mathbf{DFT}_m \otimes \mathrm{I}_{n/p})\right)\left((\mathsf{L}^{mp}_p \otimes \mathrm{I}_{n/p\mu}) \otimes_\mu \mathrm{I}_\mu\right)$$
$$\left(\bigoplus_{i=0}^{p-1}{}_\| \,\mathsf{T}^{mn,i}_n\right)\left(\mathrm{I}_p \otimes_\| (\mathrm{I}_{m/p} \otimes \mathbf{DFT}_n)\right)\left(\mathrm{I}_p \otimes_\| \mathsf{L}^{mn/p}_{m/p}\right)\left((\mathsf{L}^{pn}_p \otimes \mathrm{I}_{m/p\mu}) \otimes_\mu \mathrm{I}_\mu\right)$$

Vectorization:

$$\underbrace{\left(\overline{\mathbf{DFT}_{mn}}\right)}_{\text{vec}(\nu)} \to \underbrace{\left(\overline{(\mathbf{DFT}_m \otimes \mathrm{I}_n)\mathsf{T}^{mn}_n(\mathrm{I}_m \otimes \mathbf{DFT}_n)\,\mathsf{L}^{mn}_m}\right)}_{\text{vec}(\nu)}$$

$$\dots$$

$$\to \overleftarrow{\underbrace{(\mathrm{DFT}_m \otimes \mathrm{I}_n)}_{\text{vec}(\nu)}}^{\nu}\overline{\Big(\underbrace{\mathsf{T}^{mn}_n}_{\text{vec}(\nu)}\Big)}^{\nu}\overrightarrow{\underbrace{(\mathrm{I}_m \otimes \mathrm{DFT}_n)\,\mathsf{L}^{mn}_m}_{\text{vec}(\nu)}}^{\nu}$$

$$\dots$$

$$\to (\mathrm{I}_{mn/\nu} \otimes \underbrace{\mathsf{L}^{2\nu}_\nu}_{\text{sse}})(\overline{\mathrm{DFT}_m \otimes \mathrm{I}_{n/\nu}} \vec{\otimes} \mathrm{I}_\nu)\overline{\underbrace{(\mathsf{T}^{mn}_n)}_{\text{sse}}}^{\nu}$$
$$\left(\mathrm{I}_{m/\nu} \otimes (\overline{\mathsf{L}^n_\nu} \vec{\otimes} \mathrm{I}_\nu)(\mathrm{I}_{n/\nu} \otimes (\mathsf{L}^{2\nu}_\nu \vec{\otimes} \mathrm{I}_\nu)(\mathrm{I}_2 \otimes \underbrace{\mathsf{L}^{\nu^2}_\nu}_{\text{sse}})(\mathsf{L}^{2\nu}_2 \vec{\otimes} \mathrm{I}_\nu))(\overline{\mathrm{DFT}_n} \vec{\otimes} \mathrm{I}_\nu)\right)$$
$$\left((\mathsf{L}^{mn}_m \otimes \mathrm{I}_2) \vec{\otimes} \mathrm{I}_\nu\right)(\mathrm{I}_{mn/\nu} \otimes \underbrace{\mathsf{L}^{2\nu}_2}_{\text{sse}})$$

GPUs:

$$\underbrace{\left(\mathbf{DFT}_{r^k}\right)}_{\text{gpu}(t,c)} \to \underbrace{\left(\prod_{i=0}^{k-1} \mathsf{L}^{r^k}_r \left(\mathrm{I}_{r^{k-1}} \otimes \mathbf{DFT}_r\right)\left(\mathsf{L}^{r^k}_{r^{k-i-1}}(\mathrm{I}_{r^i} \otimes \mathsf{T}^{r^{k-i}}_{r^{k-i-1}})\,\underbrace{\mathsf{L}^{r^k}_{r^{i+1}}}_{\text{vec}(c)}\right)\right)\mathsf{R}^{r^k}_r}_{\text{gpu}(t,c)}$$

$$\dots$$

$$\to \left(\prod_{i=0}^{k-1} (\mathsf{L}^{r^n/2}_r \vec{\otimes} \mathrm{I}_2)\left(\mathrm{I}_{r^{n-1}/2} \otimes_\times \underbrace{(\mathbf{DFT}_r \vec{\otimes} \mathrm{I}_2)\,\mathsf{L}^{2r}_r}_{\text{shd}(t,c)}\right)\mathsf{T}_i\right)$$
$$(\mathsf{L}^{r^n/2}_r \vec{\otimes} \mathrm{I}_2)(\mathrm{I}_{r^{n-1}/2} \otimes_\times \underbrace{\mathsf{L}^{2r}_r}_{\text{shd}(t,c)})(\mathsf{R}^{r^{n-1}}_r \vec{\otimes} \mathrm{I}_r)$$

Verilog for FPGAs:

$$\underbrace{\left(\mathbf{DFT}_{r^k}\right)}_{\text{stream}(r^s)} \to \underbrace{\left[\prod_{i=0}^{k-1} \mathsf{L}^{r^k}_r \left(\mathrm{I}_{r^{k-1}} \otimes \mathbf{DFT}_r\right)\left(\mathsf{L}^{r^k}_{r^{k-i-1}}(\mathrm{I}_{r^i} \otimes \mathsf{T}^{r^{k-i}}_{r^{k-i-1}})\,\mathsf{L}^{r^k}_{r^{i+1}}\right)\right]\mathsf{R}^{r^k}_r}_{\text{stream}(r^s)}$$

$$\dots$$

$$\to \left[\prod_{i=0}^{k-1} \underbrace{\mathsf{L}^{r^k}_r}_{\text{stream}(r^s)} \underbrace{\left(\mathrm{I}_{r^{k-1}} \otimes \mathbf{DFT}_r\right)}_{\text{stream}(r^s)} \underbrace{\left(\mathsf{L}^{r^k}_{r^{k-i-1}}(\mathrm{I}_{r^i} \otimes \mathsf{T}^{r^{k-i}}_{r^{k-i-1}})\,\mathsf{L}^{r^k}_{r^{i+1}}\right)}_{\text{stream}(r^s)}\right] \underbrace{\mathsf{R}^{r^k}_r}_{\text{stream}(r^s)}$$

$$\dots$$

$$\to \left[\prod_{i=0}^{k-1} \underbrace{\mathsf{L}^{r^k}_r}_{\text{stream}(r^s)} \left(\mathrm{I}_{r^{k-s-1}} \otimes_s (\mathrm{I}_{r^{s-1}} \otimes \mathbf{DFT}_r)\right) \underbrace{\mathsf{T}'_i}_{\text{stream}(r^s)}\right] \underbrace{\mathsf{R}^{r^k}_r}_{\text{stream}(r^s)}$$

- Rigorous, correct by construction
- Overcomes compiler limitations

Computer generated Functions for Intel IPP

3984 C functions

1M lines of code

Transforms: DFT (fwd+inv), RDFT (fwd+inv), DCT2, DCT3, DCT4, DHT, WHT
Sizes: 2–64 (DFT, RDFT, DHT); 2-powers (DCTs, WHT)
Precision: single, double
Data type: scalar, SSE, AVX (DFT, DCT), LRB (DFT)

Computer generated

Results: SpiralGen Inc.

Challenge: General Size Libraries

So far:
Code specialized to fixed input size

```
DFT_384(x, y) {
  …
  for(i = …) {
   t[2i]   = x[2i] + x[2i+1]
   t[2i+1] = x[2i] - x[2i+1]
  }
  …
}
```

- Algorithm fixed
- Nonrecursive code

Challenge:
Library for general input size

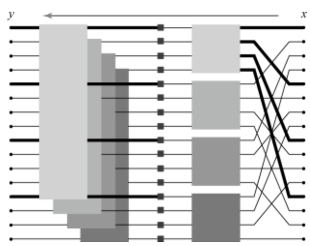
```
DFT(n, x, y) {
  …
  for(i = …) {
    DFT_strided(m, x+mi, y+i, 1, k)
  }
  …
}
```

- Algorithm cannot be fixed
- Recursive code
- Creates many challenges

Challenge: Recursive Steps Needed

Cooley-Tukey FFT

$$y = (\mathbf{DFT}_k \otimes I_m) T^{km}_m (I_k \otimes \mathbf{DFT}_m) L^{km}_k x$$



Implementation that increases locality (e.g., FFTW 2.x)

```
void DFT(int n, cpx *y, cpx *x) {
  int k = choose_dft_radix(n);

  for (int i=0; i < k; ++i)
    DFTrec(m, y + m*i, x + i, k, 1);
  for (int j=0; j < m; ++j)
    DFTscaled(k, y + j, t[j], m);
}
```

Σ–SPL : Basic Idea

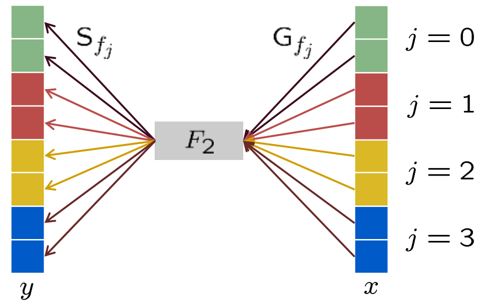
Four additional matrix constructs: Σ, G, S, Perm

Σ (sum)	*matrix sum (explicit loop)*
G_f (gather)	*load data with index mapping f*
S_f (scatter)	*store data with index mapping f*
$Perm_f$	*permute data with the index mapping f*

Σ-SPL formulas = matrix factorizations

Example: $y = (I_4 \otimes F_2)x \;\rightarrow\; y = \sum_{j=0}^{3} \mathsf{S}_{f_j} F_2 \, \mathsf{G}_{f_j} x$

$$y = \begin{bmatrix} F_2 & & & \\ & F_2 & & \\ & & F_2 & \\ & & & F_2 \end{bmatrix} x$$



Find Recursion Step Closure

Voronenko, thesis 2008

$$\{\mathbf{DFT}_n\}$$

$$\downarrow$$

$$(\{\mathbf{DFT}_{n/k}\} \otimes I_k) T_k^n (I_{n/k} \otimes \{\mathbf{DFT}_k\}) L_{n/k}^n$$

$$\downarrow$$

$$\left(\sum_{i=0}^{k-1} \mathsf{S}_{h_{i,k}} \{\mathbf{DFT}_{n/k}\} \, \mathsf{G}_{h_{i,k}}\right) \mathrm{diag}(f) \left(\sum_{j=0}^{n/k-1} \mathsf{S}_{h_{jk,1}} \{\mathbf{DFT}_k\} \, \mathsf{G}_{h_{jk,1}}\right) \mathrm{perm}(\ell_{n/k}^n)$$

$$\downarrow$$

$$\sum_{i=0}^{k-1} \mathsf{S}_{h_{i,k}} \{\mathbf{DFT}_{n/k}\} \, \mathrm{diag}(f \circ h_{i,k}) \, \mathsf{G}_{h_{i,k}} \sum_{j=0}^{n/k-1} \mathsf{S}_{h_{jk,1}} \{\mathbf{DFT}_k\} \, \mathsf{G}_{h_{j,n/k}}$$

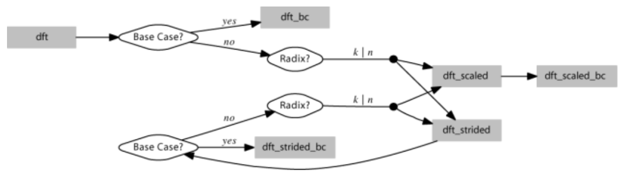
$$\downarrow$$

$$\sum_{i=0}^{k-1} \left\{ \mathsf{S}_{h_{i,k}} \mathbf{DFT}_{n/k} \, \mathrm{diag}(f \circ h_{i,k}) \, \mathsf{G}_{h_{i,k}} \right\} \sum_{j=0}^{n/k-1} \left\{ \mathsf{S}_{h_{jk,1}} \mathbf{DFT}_k \, \mathsf{G}_{h_{j,n/k}} \right\}$$

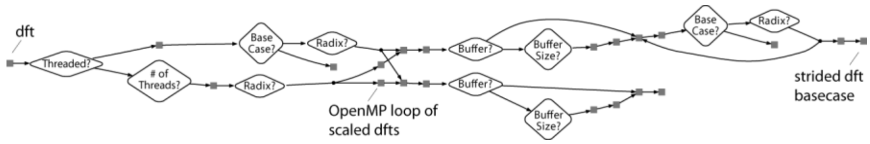
Repeat until closure

Recursion Step Closure: Examples

DFT: scalar code



DFT: full-fledged (vectorized and parallel code)



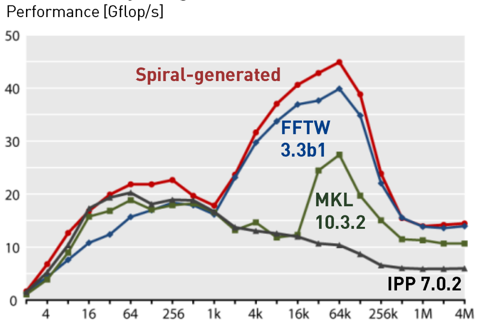
Generating Dozens of "FFTWs"

$$
\begin{aligned}
\mathbf{DFT}_n &\to P^{\top}_{k/2,2m}\left(\mathbf{DFT}_{2m} \oplus \left(I_{k/2-1} \otimes_i C_{2m}\,\mathbf{rDFT}_{2m}(i/k)\right)\right)\left(\mathbf{RDFT}'_k \otimes I_m\right), \quad k \text{ even},\\
\begin{vmatrix}\mathbf{RDFT}_n\\ \mathbf{RDFT}'_n\\ \mathbf{DHT}_n\\ \mathbf{DHT}'_n\end{vmatrix} &\to (P^{\top}_{k/2,m} \otimes I_2)\left(\begin{pmatrix}\mathbf{RDFT}_{2m}\\ \mathbf{RDFT}'_{2m}\\ \mathbf{DHT}_{2m}\\ \mathbf{DHT}'_{2m}\end{pmatrix} \oplus \left(I_{k/2-1} \otimes_i D_{2m}\begin{vmatrix}\mathbf{rDFT}_{2m}(i/k)\\ \mathbf{rDFT}_{2m}(i/k)\\ \mathbf{rDHT}_{2m}(i/k)\\ \mathbf{rDHT}_{2m}(i/k)\end{vmatrix}\right)\right)\left(\begin{vmatrix}\mathbf{RDFT}'_k\\ \mathbf{RDFT}'_k\\ \mathbf{DHT}'_k\\ \mathbf{DHT}'_k\end{vmatrix} \otimes I_m\right), \quad k \text{ even},\\
\begin{matrix}\mathbf{rDFT}_{2n}(u)\\ \mathbf{rDHT}_{2n}(u)\end{matrix} &\to L^{2n}_m\left(I_k \otimes_i \begin{matrix}\mathbf{rDFT}_{2m}((i+u)/k)\\ \mathbf{rDHT}_{2m}((i+u)/k)\end{matrix}\right)\left(\begin{vmatrix}\mathbf{rDFT}_{2k}(u)\\ \mathbf{rDHT}_{2k}(u)\end{vmatrix} \otimes I_m\right),\\
\mathbf{RDFT\text{-}3}_n &\to (Q^{\top}_{k/2,m} \otimes I_2)\left(I_k \otimes_i \mathbf{rDFT}_{2m})(i+1/2)/k)\right)(\mathbf{RDFT\text{-}3}_k \otimes I_m), \quad k \text{ even},\\
\mathbf{DCT\text{-}2}_n &\to P^{\top}_{k/2,2m}\left(\mathbf{DCT\text{-}2}_{2m} K^{2m}_2 \oplus \left(I_{k/2-1} \otimes N_{2m}\,\mathbf{RDFT\text{-}3}^{\top}_{2m}\right)\right) B_n (L^{n/2}_{k/2} \otimes I_2)(I_m \otimes \mathbf{RDFT}'_k) Q_{m/2,k},\\
\mathbf{DCT\text{-}3}_n &\to \mathbf{DCT\text{-}2}^{\top}_n,\\
\mathbf{DCT\text{-}4}_n &\to Q^{\top}_{k/2,2m}\left(I_{k/2} \otimes N_{2m}\,\mathbf{RDFT\text{-}3}^{\top}_{2m}\right) B'_n (L^{n/2}_{k/2} \otimes I_2)(I_m \otimes \mathbf{RDFT\text{-}3}_k) Q_{m/2,k},\\
\mathbf{DFT}_n &\to (\mathbf{DFT}_k \otimes \mathrm{I}_m)\,\mathrm{T}^n_m (\mathrm{I}_k \otimes \mathbf{DFT}_m)\,\mathrm{L}^n_k, \quad n = km\\
\mathbf{DFT}_n &\to P_n(\mathbf{DFT}_k \otimes \mathbf{DFT}_m) Q_n, \quad n = km,\ \gcd(k,m) = 1\\
\mathbf{DFT}_p &\to R^T_p(\mathrm{I}_1 \oplus \mathbf{DFT}_{p-1}) D_p (\mathrm{I}_1 \oplus \mathbf{DFT}_{p-1}) R_p, \quad p \text{ prime}\\
\mathbf{DCT\text{-}3}_n &\to (\mathrm{I}_m \oplus \mathrm{J}_m)\,\mathrm{L}^n_m(\mathbf{DCT\text{-}3}_m(1/4) \oplus \mathbf{DCT\text{-}3}_m(3/4))\\
&\qquad \cdot (\mathrm{F}_2 \otimes \mathrm{I}_m)\begin{bmatrix}\mathrm{I}_m & 0 \oplus -\mathrm{J}_{m-1}\\ & \frac{1}{\sqrt{2}}(\mathrm{I}_1 \oplus 2\,\mathrm{I}_m)\end{bmatrix}, \quad n = 2m\\
\mathbf{DCT\text{-}4}_n &\to S_n \mathbf{DCT\text{-}2}_n\,\mathrm{diag}_{0 \le k < n}(1/(2\cos((2k+1)\pi/4n)))\\
\mathbf{IMDCT}_{2m} &\to (\mathrm{J}_m \oplus \mathrm{I}_m \oplus \mathrm{I}_m \oplus \mathrm{J}_m)\left(\left(\begin{bmatrix}1\\1\end{bmatrix} \otimes \mathrm{I}_m\right) \oplus \left(\begin{bmatrix}-1\\1\end{bmatrix} \otimes \mathrm{I}_m\right)\right)\mathrm{J}_{2m}\,\mathbf{DCT\text{-}4}_{2m}\\
\mathbf{WHT}_{2^k} &\to \prod_{i=1}^{t}(\mathrm{I}_{2^{k_1+\cdots+k_{i-1}}} \otimes \mathbf{WHT}_{2^{k_i}} \otimes \mathrm{I}_{2^{k_{i+1}+\cdots+k_t}}), \quad k = k_1 + \cdots + k_t\\
\mathbf{DFT}_2 &\to \mathrm{F}_2\\
\mathbf{DCT\text{-}2}_2 &\to \mathrm{diag}(1, 1/\sqrt{2})\,\mathrm{F}_2\\
\mathbf{DCT\text{-}4}_2 &\to \mathrm{J}_2 \mathrm{R}_{13\pi/8}
\end{aligned}
$$

Transform	Code size	
	non-parallelized	parallelized
no vectorization		
DFT	13.1 KLOC / 0.59 MB	10.3 KLOC / 0.45 MB
RDFT	8.5 KLOC / 0.36 MB	8.8 KLOC / 0.39 MB
DHT	9.1 KLOC / 0.40 MB	9.4 KLOC / 0.39 MB
DCT-2	12.0 KLOC / 0.55 MB	12.4 KLOC / 0.57 MB
DCT-3	12.0 KLOC / 0.56 MB	12.3 KLOC / 0.59 MB
DCT-4	6.8 KLOC / 0.33 MB	7.1 KLOC / 0.35 MB
WHT	5.6 KLOC / 0.21 MB	—
2-way vectorization		
DFT	14.8 KLOC / 0.73 MB	15.0 KLOC / 0.74 MB
RDFT	15.6 KLOC / 0.76 MB	16.0 KLOC / 0.81 MB
scaled RDFT	16.0 KLOC / 0.78 MB	—
DHT	16.9 KLOC / 0.83 MB	17.2 KLOC / 0.87 MB
DCT-2	20.7 KLOC / 1.10 MB	21.0 KLOC / 1.09 MB
DCT-3	27.9 KLOC / 1.56 MB	28.2 KLOC / 1.59 MB
DCT-4	7.8 KLOC / 0.47 MB	8.1 KLOC / 0.50 MB
WHT	6.9 KLOC / 0.32 MB	5.8 KLOC / 0.26 MB
FIR Filter	167 KLOC / 7.75 MB	120 KLOC / 5.12 MB
Downsampled FIR Filter	100 KLOC / 4.2 MB	68 KLOC / 2.76 MB
4-way vectorization		
DFT	17.9 KLOC / 1.09 MB	18.2 KLOC / 1.11 MB
RDFT	16.2 KLOC / 0.86 MB	16.5 KLOC / 0.91 MB
scaled RDFT	16.5 KLOC / 0.88 MB	—
DHT	17.9 KLOC / 1.02 MB	18.3 KLOC / 1.04 MB
DCT-2	23.3 KLOC / 1.50 MB	23.6 KLOC / 1.53 MB
DCT-3	32.0 KLOC / 2.17 MB	32.3 KLOC / 2.20 MB
DCT-4	8.3 KLOC / 0.63 MB	8.6 KLOC / 0.66 MB
WHT	8.5 KLOC / 0.53 MB	6.9 KLOC / 0.4 MB
2D DFT	20.6 KLOC / 1.32 MB	20.8 KLOC / 1.33 MB
2D DCT-2	27.0 KLOC / 2.1 MB	27.2 KLOC / 2.11 MB
FIR Filter	109 KLOC / 5.69 MB	74 KLOC / 3.44 MB
Downsampled FIR Filter	151 KLOC / 7.7 MB	92 KLOC / 4.61 MB

It Really Works

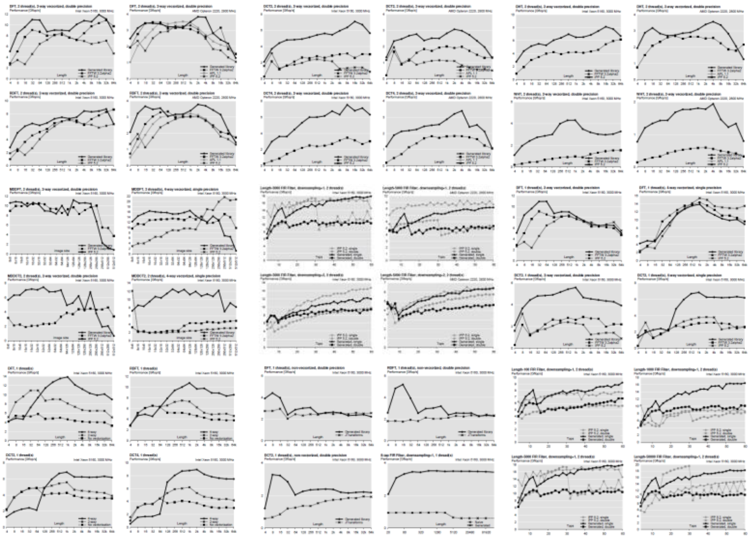
DFT on Sandybridge (3.3 GHz, 4 Cores, AVX)



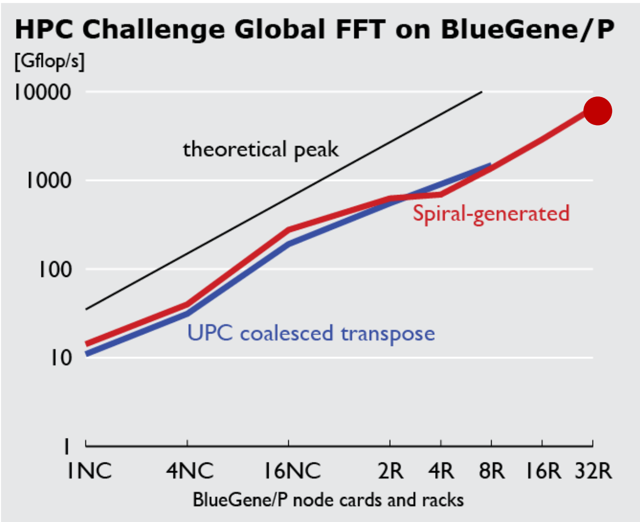
$$\mathbf{DFT}_n \to (\mathbf{DFT}_k \otimes I_m) T^n_m (I_k \otimes \mathbf{DFT}_m) L^n_k$$
$$\mathbf{DFT}_n \to P^\top_{k/2,2m} \left(\mathbf{DFT}_{2m} \oplus \left(I_{k/2-1} \otimes_i C_{2m}\, \mathbf{rDFT}_{2m}(i/k)\right)\right) (\mathbf{RDFT}_k \otimes I_m)$$
$$\mathbf{RDFT}_n \to (P^\top_{k/2,m} \otimes I_2) \left(\mathbf{RDFT}_{2m} \oplus \left(I_{k/2-1} \otimes_i D_{2m}\, \mathbf{rDFT}_{2m}(i/k)\right)\right) (\mathbf{RDFT}_k \otimes I_m)$$
$$\mathbf{rDFT}_{2n}(u) \to L^{2n}_m (I_k \otimes_i \mathbf{rDFT}_{2m}((i+u)/k)) (\mathbf{rDFT}_{2k}(u) \otimes I_m)$$

vectorized, threaded, platform-tuned, adaptive library (5 MB source code)

Generating Dozens of "FFTWs"



Very Large Scale: BG/P



6.4 Tflop/s

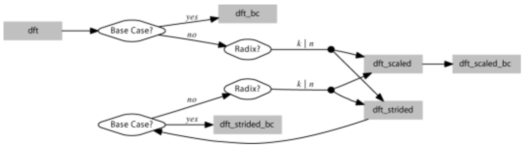
32 racks
= 32K node cards
= 128K cores

Is everything automated?

Yes, except efficient tuning

32

Simple Generated Library (~FFTW 2.x)

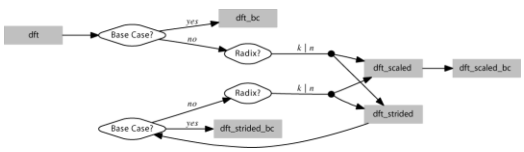



```
void dft(int n, cpx *y, cpx *x) {
    if (use_dft_base_case(n))
        dft_bc(n, y, x);
    else {
        int k = choose_dft_radix(n);
        for (int i=0; i < k; ++i)
            dft_strided(m, k, t + m*i, x + m*i);
        for (int i=0; i < m; ++i)
            dft_scaled(k, m, precomp_d[i], y + i, t + i);
    }
}

void dft_strided(int n, int istr, cpx *y, cpx *x) { ... }
void dft_scaled(int n, int str, cpx *d, cpx *y, cpx *x) { ... }
```

Choices used for tuning/adaptation

Recursion

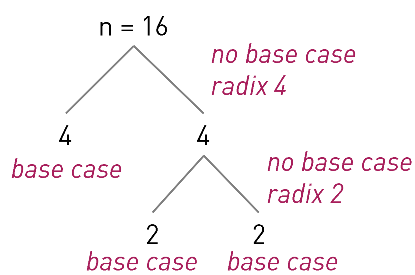
FFT Search Space (Simple Library)



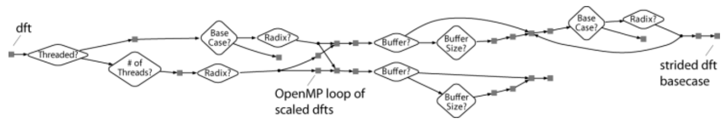
Recursive choice:

$n = 2^k$ *base case?* *radix?*

Example selections for n = 16:



FFT Search Space (The Real Thing)

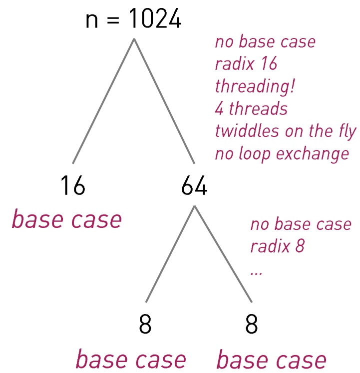


Recursive choice:

$n = 2^k$

base case?
radix?
threading?
#threads?
twiddles?
loop exchange?

Example selections for n = 1024:



When to Tune?

machine learning

offline tuning

Time of installation
(platform known)

online tuning

Time of use
(input known)

Generated FFT library

```
d = dft(n)
d(x,y)
```

compute twiddle factors
search for best recursion

user waits ...

When to Tune?

offline tuning

Time of installation
(platform known)

Generated FFT library

```
d = dft(n1)
d = dft(n2)
…
```

search for best recursion for a few n

learn decision trees

Time of use
(input known)

Generated FFT library

```
d = dft(n)
d(x,y)
```

compute twiddle factors
no search

Online Tuning → Offline Tuning

At installation time, run search for a few n

Learn decision trees

Insert into library

```
void dft(int n, cpx *y, cpx *x) {
    if (use_dft_base_case(n))
        dft_bc(n, y, x);
    else {
        int k = choose_dft_radix(n);
        for (int i=0; i < k; ++i)
            dft_strided(m, k, t + m*i, x + m*i);
        for (int i=0; i < m; ++i)
            dft_scaled(k, m, precomp_d[i], y + i, t + i);
    }
}
void dft_strided(int n, int istr, cpx *y, cpx *x) { ... }
void dft_scaled(int n, int str, cpx *d, cpx *y, cpx *x) { ... }
```

Choices used for tuning/adaptation

Online Tuning → Offline Tuning

At installation time, run search for a few n

Learn decision trees

Insert into library

Example decision tree

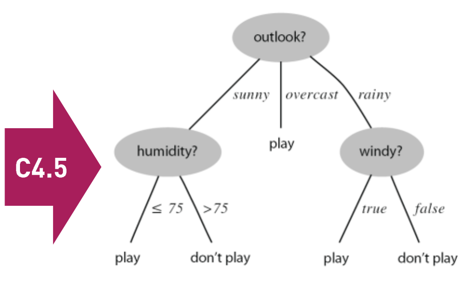
```
if ( n <= 65536 ) {
   if ( n <= 32 ) {
      if ( n <= 4 ) {return 2;}
      else {return 4;}
   }
   else {
      if ( n <= 1024 ) {
         if ( n <= 256 ) {return 8;}
         else {return 32;}
      }
      else {

   ......................................
```

```
void dft(int n, cpx *y, cpx *x) {
    if (use_dft_base_case(n))
        dft_bc(n, y, x);
    else {
        int k = choose_dft_radix(n);
        for (int i=0; i < k; ++i)
            dft_strided(m, k, t + m*i, x + m*i);
        for (int i=0; i < m; ++i)
            dft_scaled(k, m, precomp_d[i], y + i, t + i);
    }
}
void dft_strided(int n, int istr, cpx *y, cpx *x) { ... }
void dft_scaled(int n, int str, cpx *d, cpx *y, cpx *x) { ... }
```

Decision Tree Generation: C4.5

Features (events)

Outlook	Temperature	Humidity	Windy	Decision
sunny	85	85	false	don't play
sunny	80	90	true	don't play
overcast	83	78	false	play
rain	70	96	false	play
rain	68	80	false	play
rain	65	70	true	don't play
overcast	64	65	true	play
sunny	72	95	false	don't play
sunny	69	70	false	play
rain	75	80	false	play
sunny	75	70	true	play
overcast	72	90	true	play
overcast	81	75	false	play
rain	71	80	true	don't play



P(play|windy=false) = 6/8

P(don't play|windy=false) = 2/8

P(play|windy=true) = 1/2

P(don't play|windy=false) = 1/2

Entropy of Features

H(windy) = 0.89

H(outlook) = 0.69

H(humidity) = ...

Application to Libraries

Features = arguments of functions (except variable pointers)

```
dft(int n, cpx *y, cpx *x)
dft_strided(int n, int istr, cpx *y, cpx *x)
dft_scaled(int n, int str, cpx *d, cpx *y, cpx *x)
etc.
```

Experimental Setup

3GHz Intel Xeon 5160 (2 Core 2 Duos = 4 cores)

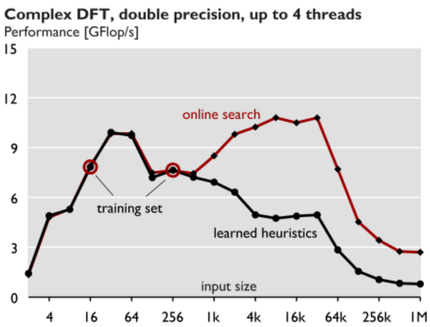
icc 10.1

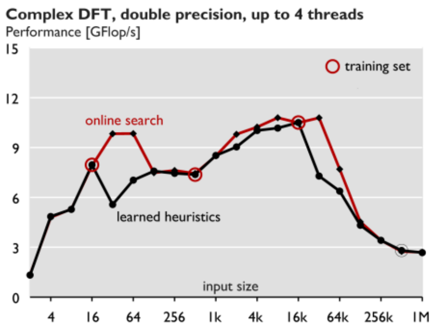
Spiral-generated "FFTW" [Voronenko et al., CGO, 2009]:

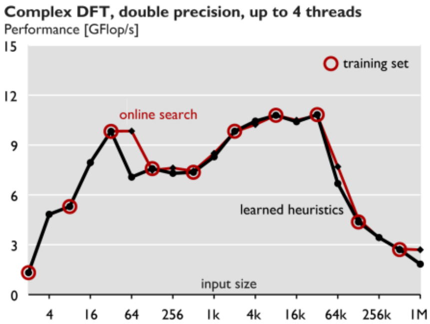
Recursive choice:

$n = 2^k$

base case?
radix?
threading?
#threads?
twiddles?
loop exchange?

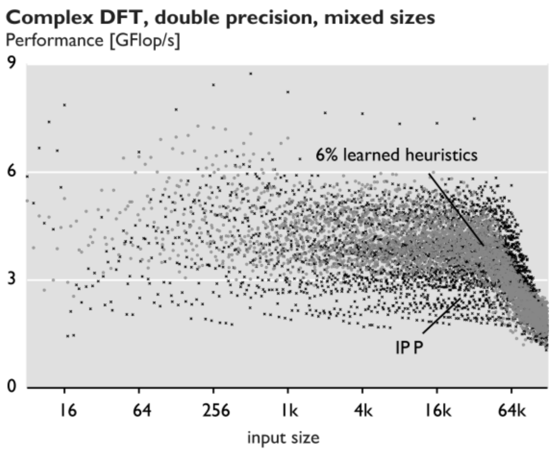






Learning works as expected

"All" Sizes



All sizes $n \le 2^{18}$, with prime factors ≤ 19

“All” Sizes

$$\mathbf{DFT}_n \rightarrow (\mathbf{DFT}_k \otimes I_m) T^n_m (I_k \otimes \mathbf{DFT}_m) L^n_k$$
$$\mathbf{DFT}_n \rightarrow P^\top_{k/2,2m} \left(\mathbf{DFT}_{2m} \oplus \left(I_{k/2-1} \otimes_i C_{2m}\, \mathbf{rDFT}_{2m}(i/k)\right)\right) (\mathbf{RDFT}_k \otimes I_m)$$
$$\mathbf{RDFT}_n \rightarrow (P^\top_{k/2,m} \otimes I_2) \left(\mathbf{RDFT}_{2m} \oplus \left(I_{k/2-1} \otimes_i D_{2m}\, \mathbf{rDFT}_{2m}(i/k)\right)\right) (\mathbf{RDFT}_k \otimes I_m)$$
$$\mathbf{rDFT}_{2n}(u) \rightarrow L^{2n}_m (I_k \otimes_i \mathbf{rDFT}_{2m}((i+u)/k)) (\mathbf{rDFT}_{2k}(u) \otimes I_m)$$

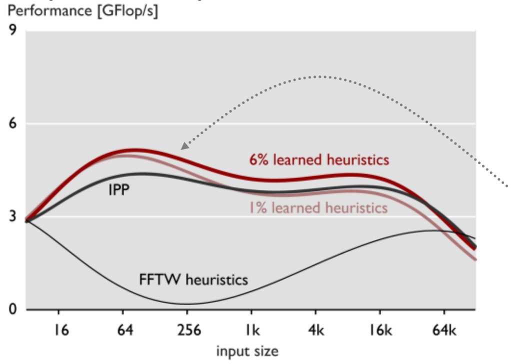
Spiral

vectorized, threaded, platform-tuned, adaptive library (5 MB source code)

Learning

vectorized, threaded, platform-tuned, adapted library (5 MB source code)

Complex DFT, double precision, mixed sizes
Performance [GFlop/s]

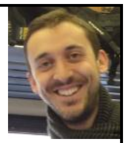


All sizes $n \leq 2^{18}$, with prime factors ≤ 19

Higher order fit of all sizes

FFT & Co. are solved, what now?

1. Generators for more domains
2. Support for building such generators



LGen: Generator for Basic Linear Algebra

Spampinato & P, CGO 2014

BLAC $y = x^T(A+B)y + \delta$

Algorithm: Tiling decision and propagation
(LL) $\left[y = x^T(A+B)y + \delta\right]_{2,3}$

vectorization

Algorithm
(-LL) $\sum_{i,j,i',j'} S_i S_{i'} \left(G_{i'} G_i A G_j G_{j'}\right)\left(G_{j'} G_j x\right)\ldots$

locality optimization

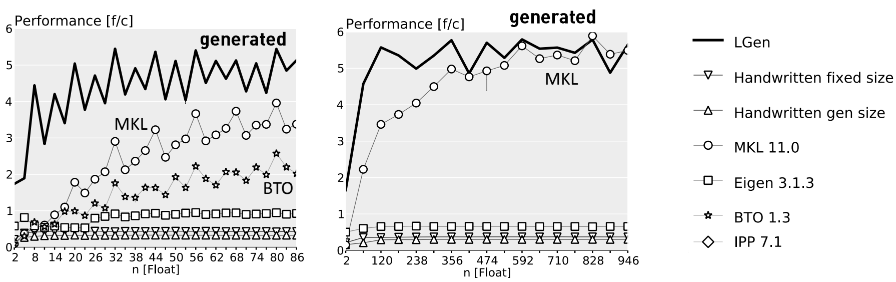
C Program

```
void kernel(float *x, float *A, float *B, …) {
float t0_64_0, t0_64_1, t0_64_2, t0_64_3 …;
    t0_57_0 = A[0];
    t0_56_0 = A[1];
    …
    t0_59_0 = t0_57_0 + t0_33_0;
    t0_63_0 = t0_59_0 * t0_9_0;
    t0_59_1 = t0_56_0 + t0_32_0;
    t0_60_0 = t0_59_1 * t0_8_0;
    < many more lines>
```

code style
code level optimization

LGen: Sample Results

$C = \alpha AB + \beta C$ $\qquad$ $C = \alpha(A_0 + A_1)^T B + \beta C$

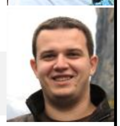


$A \in \mathbb{R}^{n\times 4}$ $\qquad$ $A_0 \in \mathbb{R}^{4\times 4}$

$B \in \mathbb{R}^{4\times n}$ $\qquad$ $B \in \mathbb{R}^{4\times n}$

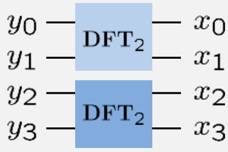
PL Support: Example Code Style

Ofenbeck, Rompf, Stojanov, Odersky & P, GPCE 2012



SPL $y = (\mathrm{I}_2 \otimes \mathbf{DFT}_2)x$

Data flow graph



Scala function

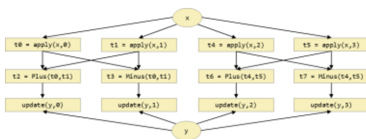
```
def f(x: Array[Double], y: Array[Double]) = {
   for (i <- 0 until 2) {
     y(2*i)   = x(i*2) + x(i*2+1)
     y(2*i+1) = x(i*2) - x(i*2+1)
   }
 }
```

```
def f(x: Array[Rep[Double]],
      y: Array[Rep[Double]]) = {
      for (i <- 0 until 2) {
        y(2*i)   = x(i*2) + x(i*2+1)
        y(2*i+1) = x(i*2) - x(i*2+1)
      }
    }
```

scalarized

```
t0 = s0 + s1;
t1 = s0 - s1;
t2 = s2 + s3;
t2 = s2 - s3;
```

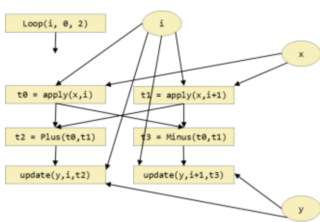
```
def f(x: Rep[Array[Double]],
      y: Rep[Array[Double]]) = {
      for (i <- 0 until 2) {
        y(2*i)   = x(i*2) + x(i*2+1)
        y(2*i+1) = x(i*2) - x(i*2+1)
      }
    }
```



unrolled, scalar repl.

```
t0 = x[0];
t1 = x[1];
t2 = t0 + t1;
y[0] = t2;
t3 = t0 - t1;
y[1] = t3;
t4 = x[0];
t5 = x[1];
t6 = t4 + x5;
y[0] = t6;
t7 = t4 – x5;
y[3] = t7;
```

```
def f(x: Rep[Array[Double]],
      y: Rep[Array[Double]]) = {
      for (i <- 0 until 2: Rep[Range]) {
        y(2*i)   = x(i*2) + x(i*2+1)
        y(2*i+1) = x(i*2) - x(i*2+1)
      }
    }
```



looped, scalar repl.

```
for (int i=0; i < 2; i++)
{
      t0 = x[i];
      t1 = x[i+1];
      t2 = t0 + t1;
      y[i] = t2;
      t3 = t0 - t1;
      y[i+1] = t3;
}
```

```
def f(x: Array[Rep[Double]],
      y: Array[Rep[Double]]) = {
      for (i <- 0 until 2) {
        y(2*i)   = x(i*2) + x(i*2+1)
        y(2*i+1) = x(i*2) - x(i*2+1)
      }
```


scalarized

```
t0 = s0 + s1;
t1 = s0 - s1;
t2 = s2 + s3;
t2 = s2 - s3;
```

Staging enables program generation

Abstracting over code style = abstracting over staging decisions

```
def f[L[_],A[_],T](looptype: L, x: A[Array[T]], y: A[Array[T]]) = {
      for (i <- 0 until 2: L[Range]) {
        y(2*i)  = x(i*2) + x(i*2+1)
        y(2*i+1)= x(i*2) - x(i*2+1)
      }
}
```

```
  for (i <- 0 until 2: Rep[Range]) {
    y(2*i)   = x(i*2) + x(i*2+1)
    y(2*i+1) = x(i*2) - x(i*2+1)
  }
}
```



```
  t1 = x[i+1];
  t2 = t0 + t1;
  y[i] = t2;
  t3 = t0 - t1;
  y[i+1] = t3;
}
```

Related Work

Program generators for performance

- *FFTW codelet generator (Frigo)*
- *Flame (van de GeijnQuinta, na-Orti, Bientinesi, …)*
- *cvxgen (Mattingley, Boyd)*
- *PetaBricks (Ansel, Amarasinghe, …)*
- *Spiral*

Autotuning

- *ATLAS/PhiPAC (Whaley, Bilmes, Demmel, Dongarra, …)*
- *FFTW adaptive library (Frigo, Johnson)*
- *OSKI (Vuduc et al.)*
- *Adaptive sorting (Li et al.)*

Environments for DSLs and program generation

- *Scala and lightweight modular staging (LMS)*
- *More examples*

Automatically from Math to Fast Code

Principles

Generate Code

"click"

Capturing algorithm knowledge:
Mathematical DSLs

Structural optimization:
Rewriting

$$\begin{aligned}
\mathbf{DFT}_n &\to (\mathbf{DFT}_k \otimes \mathrm{I}_m)\, \mathrm{T}^n_m (\mathrm{I}_k \otimes \mathbf{DFT}_m)\, \mathrm{L}^n_k, \quad n = km \\
\mathbf{DFT}_n &\to P_n(\mathbf{DFT}_k \otimes \mathbf{DFT}_m) Q_n, \quad n = km,\ \gcd(k,m) = 1 \\
\mathbf{DCT\text{-}4}_n &\to S_n \mathbf{DCT\text{-}2}_n\, \mathrm{diag}_{0\le k<n}(1/(2\cos((2k+1)\pi/4n))) \\
\mathbf{IMDCT}_{2m} &\to (\mathrm{J}_m \oplus \mathrm{I}_m \oplus \mathrm{I}_m \oplus \mathrm{J}_m)\left(\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix} \otimes \mathrm{I}_m\right) \oplus \left(\begin{bmatrix} -1 \\ -1 \end{bmatrix} \otimes \mathrm{I}_m\right)\right) \mathrm{J}_{2m}\, \mathbf{DCT\text{-}4}_{2m}
\end{aligned}$$

Decision making:
Search and learning

$$\underbrace{A_m \otimes \mathrm{I}_n}_{\mathrm{smp}(p,\mu)} \to \underbrace{\mathrm{L}^{mn}_m}_{\mathrm{smp}(p,\mu)} \left(\mathrm{I}_p \otimes_{\parallel} (\mathrm{I}_{n/p} \otimes A_m)\right) \underbrace{\mathrm{L}^{mn}_n}_{\mathrm{smp}(p,\mu)}$$

Key Challenges

New domains (linear algebra, filters, …)

Programming language support (DSLs, staging)

More information: www.spiral.net

Further Reading

Spiral (www.spiral.net)

Markus Püschel, Franz Franchetti and Yevgen Voronenko
Spiral
in Encyclopedia of Parallel Computing, Eds. David Padua, Springer 2011

Georg Ofenbeck, Tiark Rompf, Alen Stojanov, Martin Odersky and Markus Püschel
Spiral in Scala: Towards the Systematic Construction of Generators for Performance Libraries
Proc. International Conference on Generative Programming: Concepts & Experiences (GPCE), pp. 125-134, 2013

Markus Püschel, José M. F. Moura, Jeremy Johnson, David Padua, Manuela Veloso, Bryan Singer, Jianxin Xiong, Franz Franchetti, Aca Gacic, Yevgen Voronenko, Kang Chen, Robert W. Johnson, and Nick Rizzolo
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Proceedings of the IEEE special issue on "Program Generation, Optimization, and Adaptation," Vol. 93, No. 2, 2005, pp. 232-275

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A Basic Linear Algebra Compiler
Proc. International Symposium on Code Generation and Optimization (CGO), pp. 23-32, 2014

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Proc. International Symposium on Code Generation and Optimization (CGO), pp. 117-127, 2016

Tuning through machine learning

Frédéric de Mesmay, Yevgen Voronenko and Markus Püschel
Offline Library Adaptation Using Automatically Generated Heuristics
Proc. International Parallel and Distributed Processing Symposium (IPDPS), pp. 1-10, 2010

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Bandit-Based Optimization on Graphs with Application to Library Performance Tuning
Proc. International Conference on Machine Learning (ICML), pp. 729-736, 2009

Bryan Singer and Manuela Veloso
Learning to Generate Fast Signal Processing Implementations
Proc. International Conference on Machine Learning (ICML), pp. 529-536, 2001