The Pluto Compiler and its Use for Computational Sciences

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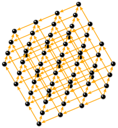

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- **Uses many other polyhedral libraries and tools like ISL, Polylib, Cloog, Pet, Clan, Candl**

How can Pluto be used?

- **Push button:** fully automatically for optimization (tiling and other transformations), parallelization
- **Almost automatic:** With an understanding of what Pluto does, use it to obtain desired result
- **DSLs**: In domain-specific compilers/optimizers or library generators

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Big Picture: Role of Compilers

- Improve existing **general-purpose** compilers (for C, C++, Python, ...)
- LLVM/Polly, GCC/Graphite, PPCG, Pluto, other compilers/tools

- Build new **domain-specific languages and compilers**
- Scientists say **WHAT** they execute and not **HOW** they execute

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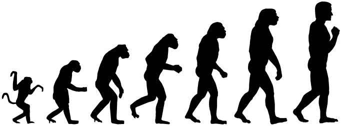

The Revolutionary Approach

- Build new **domain-specific languages and compilers**
- Scientists say **WHAT** they execute and not **HOW** they execute

Important to pursue both

OUTLINE

1. Pluto/Pluto+
   - Affine Transformations
   - Tiling
2. Case Studies
   - Solving Partial Differential Equations
   - Lattice Boltzmann Method
   - Image Processing Pipelines
3. Conclusions

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Affine Transformations

- Examples of affine functions of i, j: $i + j$, $i - j$, $i + 1$, $2i + 5$
- Not affine: ij, i^2, $i^2 + j^2$, $a[j]$

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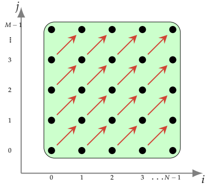


Figure: Iteration space

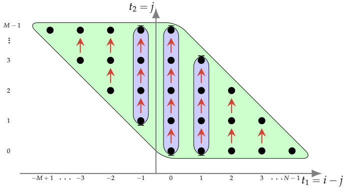


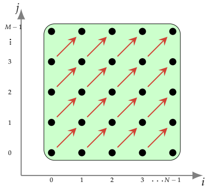
Figure: Transformed space

```
for (i = 0; i < N; i++){
    for (j = 0; j < M; j++){
          A[i+1][j+1] = f(A[i][j])
    }
}
```

```
 #pragma omp parallel for private(t2)
for (t1=-M+1; t1<=N-1; t1++) {
    for (t2=max(0,-t1); t2<=min(M-1,N-1-t1); t2++){
        A[t1+t2+1][t2+1] = f(A[t1+t2][t2]);
    }
}
```

- Transformation: $(i, j) \rightarrow (\mathbf{i-j}, \mathbf{j})$

AFFINE TRANSFORMATIONS



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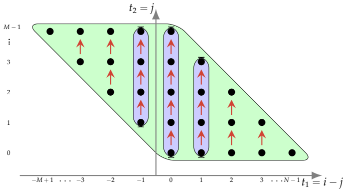

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- Affine transformations are attractive because:
  - Preserve **collinearity** of points and **ratio of distances** between points
  - Code generation with affine transformations has thus been studied well (CLooG, ISL, OMEGA+)

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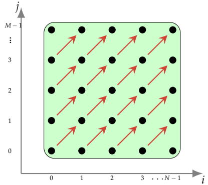

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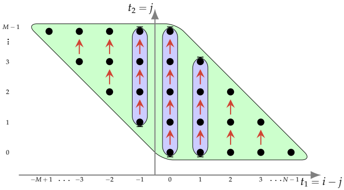

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  - Preserve **collinearity** of points and **ratio of distances** between points
  - Code generation with affine transformations has thus been studied well (CLooG, ISL, OMEGA+)
  - Model a very rich class of loop re-orderings
  - Useful for several domains like dense linear algebra, stencils, image processing pipelines, Lattice Boltzmann Method

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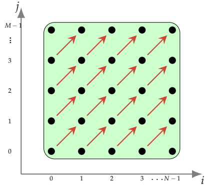

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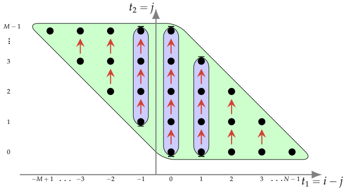

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- Affine transformations can improve **parallelism** and **locality** (Feautrier 1992, Lengauer, Lim and Lam 1997, Griebl 2004, Pluto 2008)

The Pluto Algorithm

- Designed around 2008 [Bondhugula et al. CC 2008, PLDI 2008]
- **Finds good transformations to improves locality and parallelism**
- Extended in 2014-2015 (transformation coefficients need not be non-negative)

Finding Valid and Good Affine Transformations

(i, j)
(j, i)
(i+j, j)
(i-j, j)
(i, i+j)
(i+j, i-j)
. . .

Finding Valid and Good Affine Transformations

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- One-to-one functions
- Validity: dependences should not be violated
- Coefficients: for $i - j$, the coefficients are 1,-1

Pluto Algorithm

- Optimization Problem: **Minimize dependence distance**

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Tiling (blocking)

- Partition and execute iteration space in blocks

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for (i=1; i<T; i++)
  for (j=1; j<N-1; j++)
    S(i,j)
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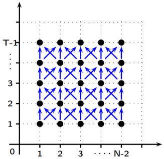

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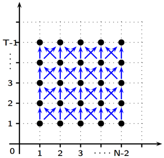

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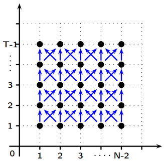

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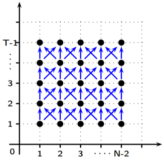

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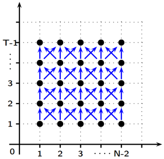

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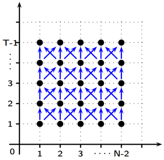

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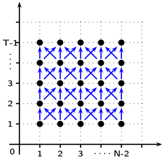

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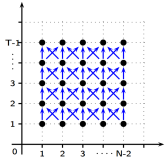

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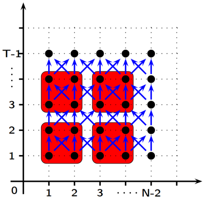

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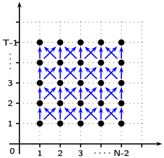

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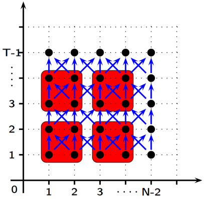

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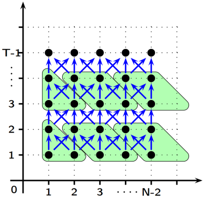

Figure: Valid tiling

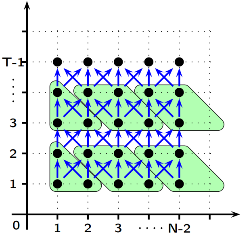

Figure: Parallelogram tiling

Concurrent Start and Diamond Tiling[1]

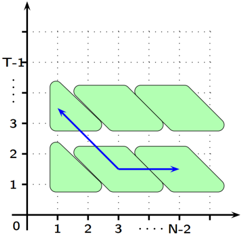

Figure: Pipelined start

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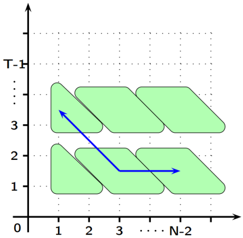

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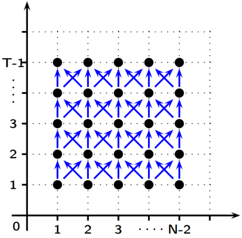

Figure: Original iteration space

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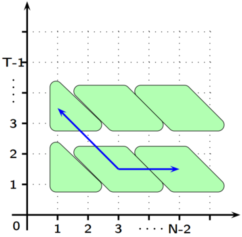

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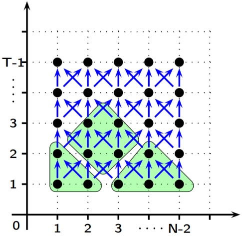

Figure: Group as diamonds

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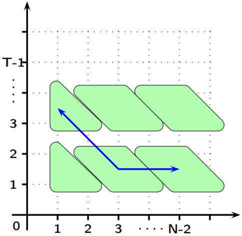

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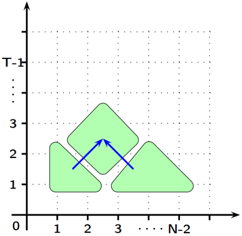

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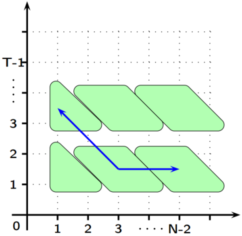

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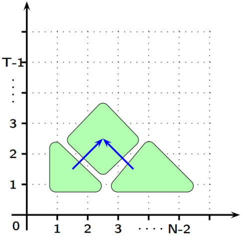

Figure: Concurrent start possible

- Diamond tiling
  - Face allowing concurrent should be strictly within the cone of the tiling hyperplanes
  - Eg: (1,0) is in the cone of (1,1) and (1,-1)

Classical Time Skewing vs Diamond Tiling

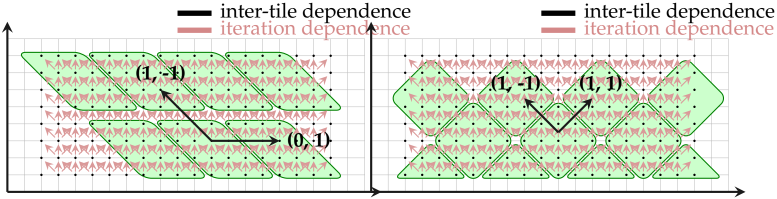

Figure: Two ways of tiling heat-1d: parallelogram & diamond

- Classical time skewing: $(t, i) \to (t, t + i)$
- Diamond tiling: $(t, i) \to (t + i, t - i)$

A sequence of transformations for 2-d Jacobi relaxations

```
for (t = 0; t < T; t++)
  for (i = 1; i < N+1; i++)
    for (j = 1; j < N+1; j++)
      A[(t+1)%2][i][j] = f((A[t%2][i+1][j], A[t%2][i][j], A[t%2][i-1][j],
                  A[t%2][i][j+1], A[t%2][i][j-1], A[t%2][i][j]);
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1. Enabling transformation for diamond tiling

$$T((t,i,j)) = (t+i, t-i, t+j).$$

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1. Enabling transformation for diamond tiling

$$T((t,i,j)) = (t+i, t-i, t+j).$$

2. Perform the actual tiling (in the transformed space)

$$T'((t,i,j)) = \left(\frac{t+i}{64}, \frac{t-i}{64}, \frac{t+j}{64}, t+i, t-i, t+j\right)$$

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3. Create a wavefront of tiles

$$T''((t,i,j)) = \left(\frac{t+i}{64} + \frac{t-i}{64}, \frac{t-i}{64}, \frac{t+j}{64}, t, t+i, t+j\right)$$

Some examples of how Pluto can be used

- Optimize Jacobi and other relaxations via time tiling

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- Optimize pre-smoothing steps at various levels of Geometric Multigrid method

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- Optimize Jacobi and other relaxations via time tiling
- Optimize pre-smoothing steps at various levels of Geometric Multigrid method
- Optimize Lattice Boltzmann Method computations

Using Pluto: Recommendations

Web: http://pluto-compiler.sf.net

- Use git version
- Use 'pet' branch of git version
- Preferable: use Intel's C/C++ compiler (14.0 or higher) to compile generated code

Outline

Poisson's Equation

Poisson's equation:

$$\nabla^2 u = f.$$

$$\frac{1}{h^2}\begin{bmatrix} & -1 & \\ -1 & 4 & -1 \\ & -1 & \end{bmatrix} u_h = f_h$$

- We are solving $y = Ax$
- What about A^{-1}?

Geometric Multigrid Method

- Use a hierarchical structure – a multi-scale representation of the grid
- Perform pre-smoothing at a finer level
- Restrict the error to a coarser grid
- Solve for the error at a coarser level (recursion)
- Interpolate the error to the finer level

- Run multiple iterations of the above

Pluto can be used to optimize the pre-smoothing or post-smoothing steps readily

Hierarchical Mesh Structure

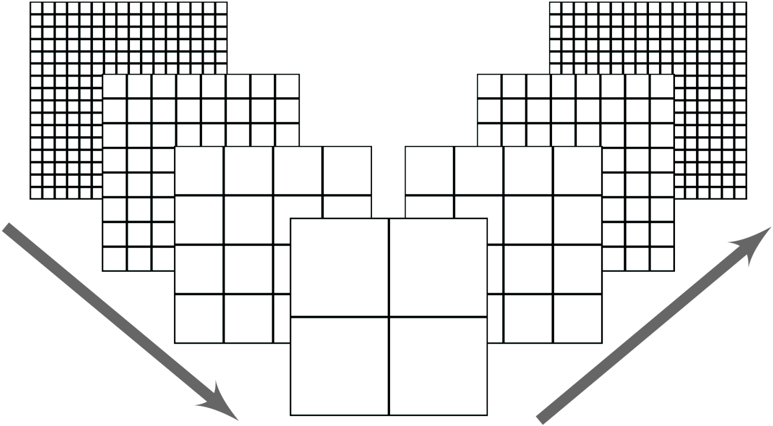


Figure: Hierarchical mesh structure for Multigrid levels

Mulitigrid V-cycle: Algorithm

Input : v^h, f^h

Relax v^h for n_1 iterations: $\nu^h \leftarrow (1 - \omega D^{-1} A^h) v^h + \omega D^{-1} f^h$
// pre-smoothing

if coarsest level **then**

  Relax v^h for n_2 iterations *// coarse smoothing*

$r^h \leftarrow f^h - A^h v^h$ *// residual*

$r^{2h} \leftarrow I_h^{2h} r^h$ *// restriction*

$e^{2h} \leftarrow 0$

$e^{2h} \leftarrow V\text{-}cycle^{2h}(e^{2h}, r^{2h})$

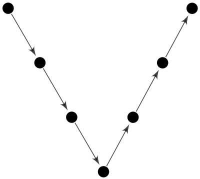
$e^h \leftarrow I_{2h}^h e^{2h}$ *// interploation*

$v^h \leftarrow v^h + e^h$ *// correction*

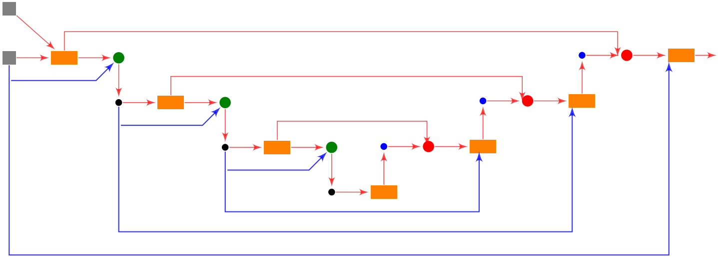
Relax v^h for n_3 iterations *// post smoothing*

return v^h

MULITIGRID V-CYCLE

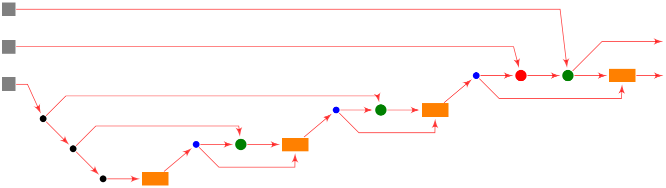


(a) V-cycle



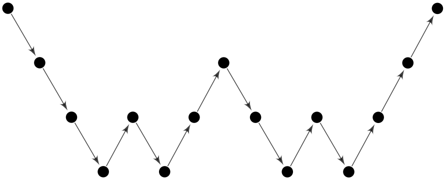
(b) V-cycle: complete DAG

NAS MG V-CYCLE

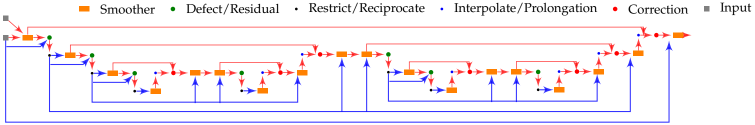


(c) NAS-PB MG V-cycle

MULTIGRID W-CYCLE



(d) W-cycle



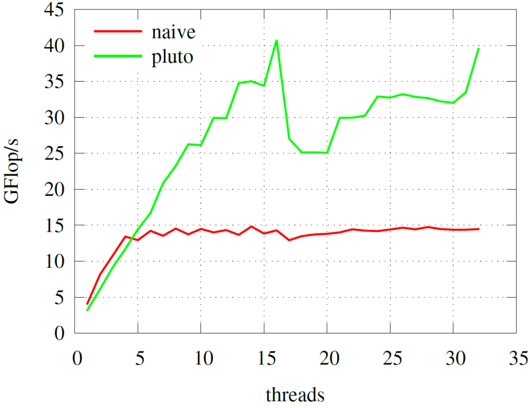
(e) W-cycle: complete DAG

Figure: DAG representation of (a) V-cycle and (b) W-cycle

Strongly recommend reading:

- P. Ghysels and W. Vanroose, Modeling the performance of geometric multigrid on many-core computer architectures, SIAM J. Scientific Computing (2015).
- W. Vanroose, P. Ghysels, D. Roose, and K.Meerbergen, Hiding global communication latency and increasing the arithmetic intensity in extreme-scale Krylov solvers, Position Paper at DOE/ASCR workshop on Applied Mathematics Research for Exascale Computing. Aug 2013.

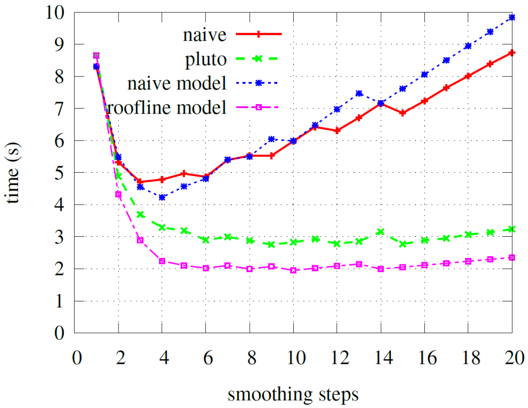
GMG: Smoother Scaling



Scalability of 10 iterations of the Jacobi smoother on an 8000^2 domain on a 16-core Intel Sandy Bridge

Source: Ghysels (LBNL) and Vanroose (University of Antwerp) SIAM J. Scientific Computing 2015

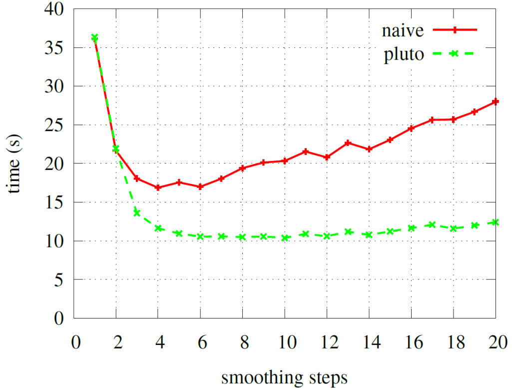
GMG: EXECUTION TIME (2-D)



Timings for a full solve on a 8191^2 domain using V -cycles with a relative stopping tolerance 10^{-12}

Source: Ghysels and Vanroose 2015

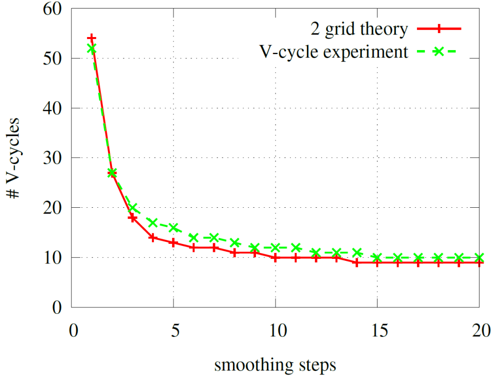
GMG: Execution time (3-d)



Timings for a full solve on a 511^3 domain using V -cycles with a relative stopping tolerance 10^{-12} on a dual socket Sandy Bridge machine for a 3D domain

Source: Ghysels and Vanroose 2015

GMG: CONVERGENCE FOR SMOOTHING STEPS



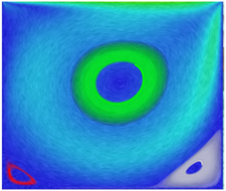
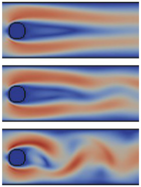
The corresponding number of V-cycles required to reach a 10^{-12} relative stopping criterion for both two-grid and multigrid.

Source: Ghysels and Vanroose 2015

Outline

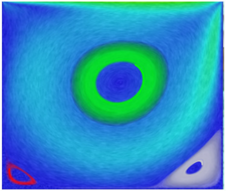
1. Pluto/Pluto+
   - Affine Transformations
   - Tiling
2. Case Studies
   - Solving Partial Differential Equations
   - Lattice Boltzmann Method
   - Image Processing Pipelines
3. Conclusions

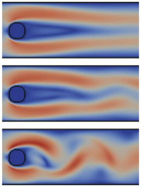
LBM: Introduction

- Lattice–Boltzmann method (LBM) is used for simulation of complex fluid flows in Computational Fluid Dynamics (CFD)

LBM: Introduction





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- The simplicity of formulation and its versatility explain the rapid expansion of LBM to applications in complex and multiscale flows

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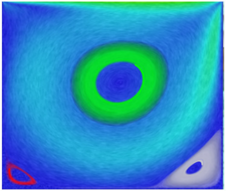
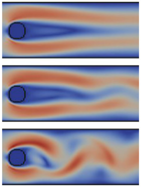

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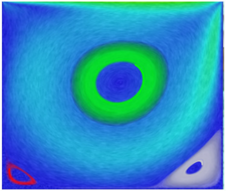

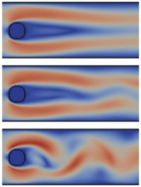

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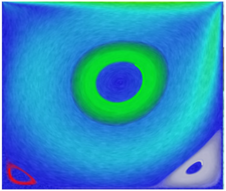
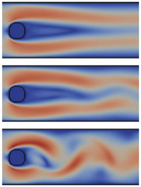
- In spite of tremendous advances in its application, several fundamental opportunities for optimization remain

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- The simplicity of formulation and its versatility explain the rapid expansion of LBM to applications in complex and multiscale flows
- Particularly suited for parallel and high performance implementations
- In spite of tremendous advances in its application, several fundamental opportunities for optimization remain
- We explore one such opportunity through this work

Lattice-Boltzmann Method

- Fluid flows are modelled as hypothetical particles
  - moving in a lattice domain (discretized space)
  - with different lattice velocities (discretized momentum)
  - over different time steps (discretized time)

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- Fluid flows are modelled as hypothetical particles
  - moving in a lattice domain (discretized space)
  - with different lattice velocities (discretized momentum)
  - over different time steps (discretized time)
- Solves the discrete Boltzmann equation for the particle distribution function (a probability density function)

LBM - Lattice Arrangements

- Lattice arrangements are represented as $DmQn$
  - m is the space dimensionality of the lattice
  - n is the number of PDFs (or speeds) involved

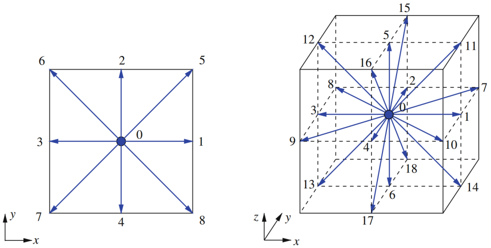

Figure: $D2Q9$ (left) & $D3Q19$ (right) lattice arrangements

LBM - Lattice Arrangements

- Lattice arrangements are represented as $DmQn$
  - m is the space dimensionality of the lattice
  - n is the number of PDFs (or speeds) involved
- Choice of lattice affects precision and duration of simulation

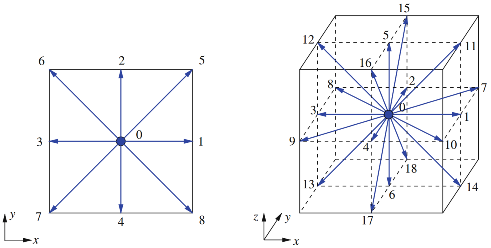

Figure: $D2Q9$ (left) & $D3Q19$ (right) lattice arrangements

- The discretized form of Lattice–Boltzmann Equation forms the basis of all LBM models

$$f_i(\mathbf{x} + \mathbf{c_i}\Delta t, t + \Delta t) = f_i(\mathbf{x}, t) + \Omega_i(f_i(\mathbf{x}, t)),\ i = 1, \ldots n. \quad (1)$$

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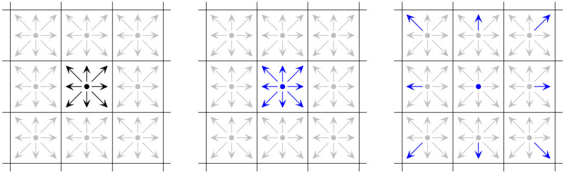
- Eqn. 1 is solved in two steps, the **collision** step (Eqn. 2) & the **advection** step (Eqn. 3)

$$f_i^*(\mathbf{x}, t + \Delta t) = f_i(\mathbf{x}, t) + \Omega_i(f_i(\mathbf{x}, t)) \quad (2)$$

$$f_i(\mathbf{x} + \mathbf{c_i}\Delta t, t + \Delta t) = f_i^*(\mathbf{x}, t + \Delta t) \quad (3)$$

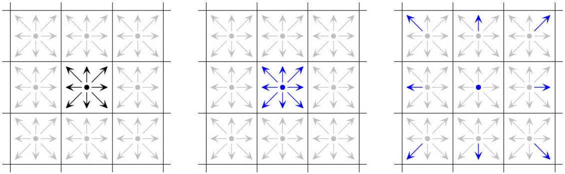
LBM - Implementation Strategies



(a) Pre-collision (b) Post-collision (c) Post-propagation

Figure: Push Scheme

LBM - Implementation Strategies



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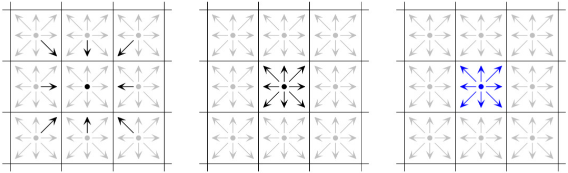

(a) Initial state (b) Pre-collision (c) Post-collision

Figure: Pull Scheme

Is it possible to optimize LBM using time tiling?

Time Tiling LBM Computations

- LBM can be written using storage for either one grid or two grids

Time Tiling LBM Computations

- LBM can be written using storage for either one grid or two grids
- One grid ⇒ Separate collision and advection
- Fused Collision + Advection ⇒ Two grids

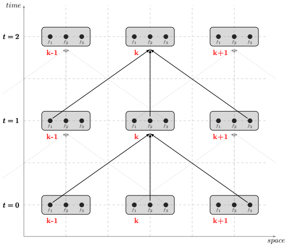

Figure: 1D LBM with single grid

1

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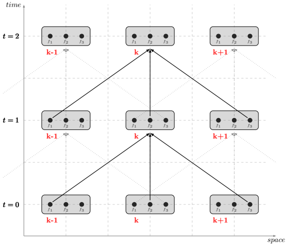

Figure: 1D LBM with single grid

- **Not possible to "*time tile*" LBM with single grid**

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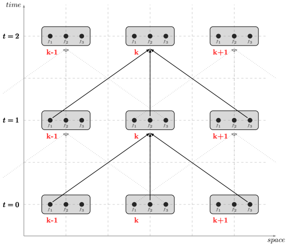

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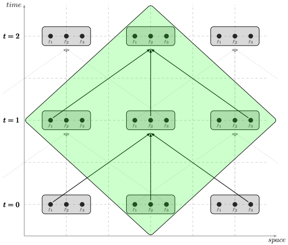

Figure: Pull scheme on a 1D LBM

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1

1

Time Tiling LBM Computations

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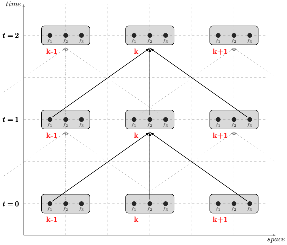

Figure: 1D LBM with single grid

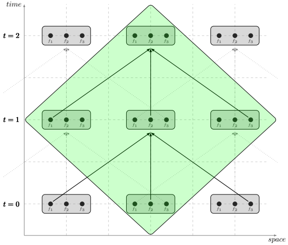

Figure: Pull scheme on a 1D LBM

- **Not possible to "*time tile*" LBM with single grid**
- **Time tiling is possible with two grids**

1

1

msLBM - LBM Optimization Framework

- We utilize a fused version of the LBM kernel
  - No explicit *advection* phase
  - 2 data grids with a pull scheme to read data
  - Array-of-Structures (*AoS*) layout for data

Input to Polyhedral Tiler

```
#pragma scop
for (t = 0; t < _nTimesteps; t++)
   for (y = 2; y < _nY; y++)
      for (x = 1; x < _nX; x++)
         lbm_kernel(grid[t % 2][y][x][C],
            grid[t % 2][y - 1][x + 0][N],
            grid[t % 2][y + 1][x + 0][S],
            grid[t % 2][y + 0][x - 1][E],
            grid[t % 2][y + 0][x + 1][W],
            grid[t % 2][y - 1][x - 1][NE],
            grid[t % 2][y - 1][x + 1][NW],
            grid[t % 2][y + 1][x - 1][SE],
            grid[t % 2][y + 1][x + 1][SW],

            &grid[(t + 1) % 2][y][x][C],
            &grid[(t + 1) % 2][y][x][N],
            &grid[(t + 1) % 2][y][x][S],
            &grid[(t + 1) % 2][y][x][E],
            &grid[(t + 1) % 2][y][x][W],
            &grid[(t + 1) % 2][y][x][NE],
            &grid[(t + 1) % 2][y][x][NW],
            &grid[(t + 1) % 2][y][x][SE],
            &grid[(t + 1) % 2][y][x][SW], t, y, x);
```

- Use the **PET polyhedral frontend [Verdoolaege and Grosser 2012]**: the LBM collision is treated as a blackbox (abstracted as a single function)
- Dependence structure is now similar to “toy time-iterated stencils”
- **All time tiling strategies can now be applied!**

Experimental Setup

Intel Xeon E5-2680 (SandyBridge)	
Clock	2.7 GHz
Cores / socket	8
Total cores	16
L1 cache / core	32 KB
L2 cache / core	512 KB
L3 cache / socket	20 MB
Peak GFLOPs	172.8
Compiler	Intel C compiler (icc) 14.0.1
Compiler flags	-O3 -xHost -ipo -fno-alias -fno-fnalias -restrict -fp-model precise -fast-transcendentals
Linux kernel	3.8.0-38

Table: Architecture details

- We compare the performance of our framework on 7 benchmarks against
  - Palabos - an open-source CFD solver based on LBM
  - Compiler auto-parallelization [*icc-auto-par*]
  - Naive manual parallelization using OpenMP [*icc-omp-par*]

BENCHMARKS

- Lid Driven Cavity - `d2q9`, `d3q19` and `d3q27`

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Benchmarks

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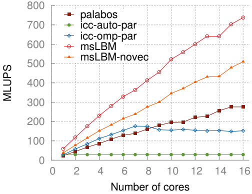
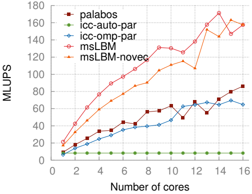
Performance Metrics

- MLUPS - Million Lattice site Updates Per Second
- MEUPS - Million Element Updates Per Second

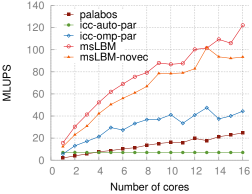
Performance - LDC



(a) `ldc-d2q9`

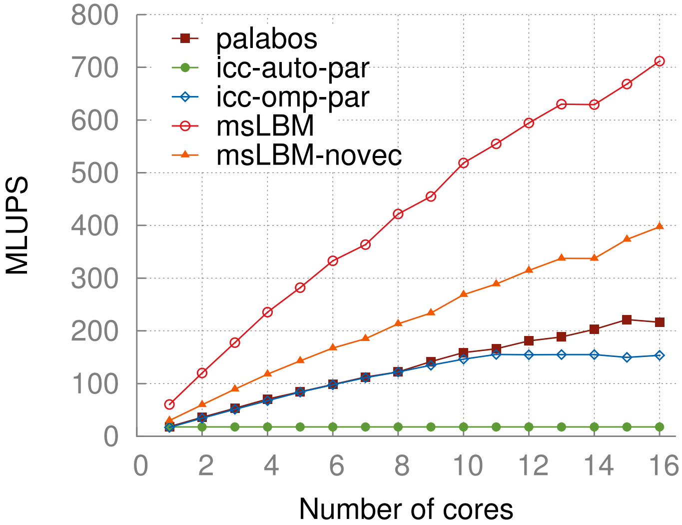


(b) `ldc-d3q19`



(c) `ldc-d3q27`

Performance - MRT (d2q9)



Roofline Performance Model - Contd.

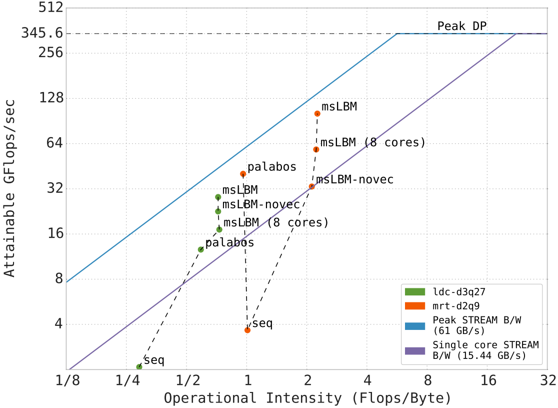


Figure: Roofline model for `mrt-d2q9` & `ldc-d3q27`

ROOFLINE PERFORMANCE MODEL - CONTD.

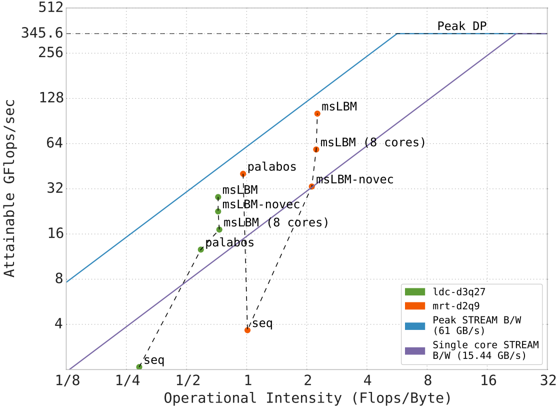


Figure: Roofline model for `mrt-d2q9` & `ldc-d3q27`

- `msLBM` obtains further improvement over Palabos in both operational intensity and peak achievable performance

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PolyMage

http://mcl.csa.iisc.ernet.in/polymage.html

A DSL and Compiler for Automatic Parallelization and Optimization of Image Processing Pipelines

Graphs of interconnected processing stages

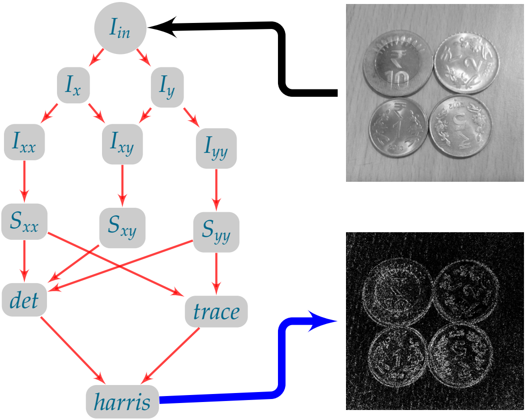
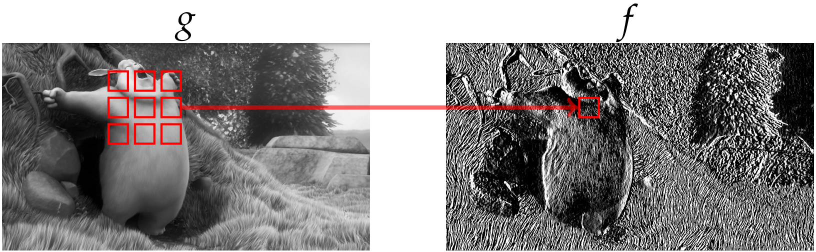


Figure: **Harris corner detection**

Point-wise

$$f(x,y) = w_r \cdot g(x,y,{\color{red}\bullet}) + w_g \cdot g(x,y,{\color{green}\bullet}) + w_b \cdot g(x,y,{\color{blue}\bullet})$$

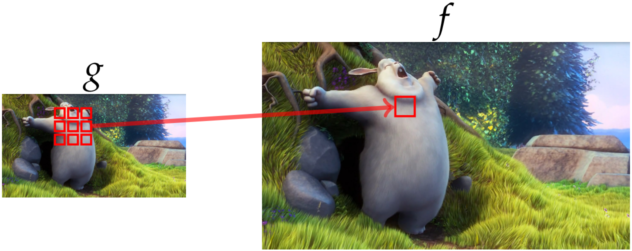


Stencil

$$f(x, y) = \sum_{\sigma_x=-1}^{+1} \sum_{\sigma_y=-1}^{+1} g(x + \sigma_x, y + \sigma_y) \cdot w(\sigma_x, \sigma_y)$$

Downsample

$$f(x,y) = \sum_{\sigma_x=-1}^{+1} \sum_{\sigma_y=-1}^{+1} g(2x+\sigma_x, 2y+\sigma_y) \cdot w(\sigma_x, \sigma_y)$$



Upsample

$$f(x,y) = \sum_{\sigma_x=-1}^{+1} \sum_{\sigma_y=-1}^{+1} g((x+\sigma_x)/2, (y+\sigma_y)/2) \cdot w(\sigma_x, \sigma_y, x, y)$$

Example: Pyramid Blending pipeline

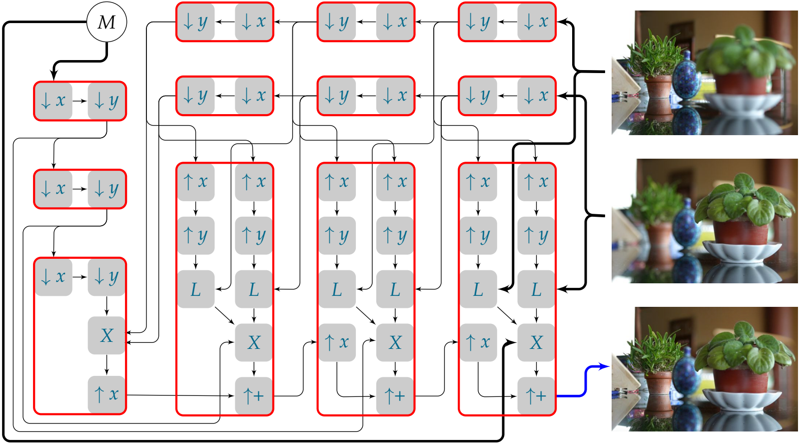


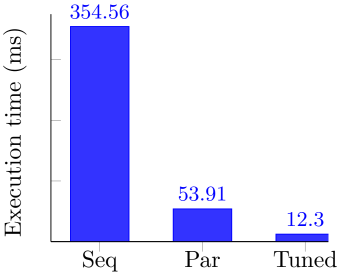
Image courtesy: Kyros Kutulakos

Where are Image Processing Pipelines used?

- On images uploaded to social networks like Facebook, Google+
- On all camera-enabled devices
- Everyday workloads from data center to mobile device scales
- **Computational photography, computer vision, medical imaging,** ...

Google+ Auto Enhance

Naive vs Optimized Implementation

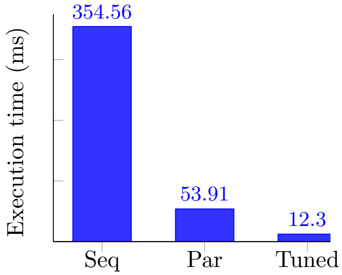


Harris corner detection (16 cores)

- Naive implementation in C
- Naive parallelization – **7**×
  OpenMP, Vector pragmas (icc)
- Manual optimization – **29**×
  Locality, Parallelism, Vector intrinsics

Manually optimizing pipelines is hard

NAIVE VS OPTIMIZED IMPLEMENTATION



Harris corner detection (16 cores)

- Naive implementation in C
- Naive parallelization – **7**×
  OpenMP, Vector pragmas (icc)
- Manual optimization – **29**×
  Locality, Parallelism, Vector intrinsics

Goal: Performance levels of manual tuning
Without the pain

Our Approach: PolyMage

- **High-level language (DSL embedded in Python)**
  - Allow expressing common patterns intuitively
  - Enables compiler analysis and optimization
- **Automatic Optimizing Code Generator**
  - Uses domain-specific cost models to apply complex combinations of scaling, alignment, **tiling** and **fusion** to optimize for **parallelism** and **locality**

Harris Corner Detection

```
R, C = Parameter(Int), Parameter(Int) (∗\label{param}∗)
I = Image(Float, [R+2, C+2]) (∗\label{image}∗)

x, y = Variable(), Variable() (∗\label{vars}∗)
row, col = Interval(0,R+1,1), Interval(0,C+1,1)(∗\label{intervals}∗)

c = Condition(x,’>=’,1) & Condition(x,’<=’,R) &(∗\label{cond1}∗)
    Condition(y,’>=’,1) & Condition(y,’<=’,C)

cb = Condition(x,’>=’,2) & Condition(x,’<=’,R-1) &(∗\label{cond2}∗)
     Condition(y,’>=’,2) & Condition(y,’<=’,C-1)

Iy = Function(varDom = ([x,y],[row,col]),Float)(∗\label{f1}∗)
Iy.defn = [ Case(c, Stencil(I(x,y), 1.0/12,(∗\label{d1}∗)
                    [[-1, -2, -1],
                     [ 0, 0, 0],
                     [ 1, 2, 1]]) ]

Ix = Function(varDom = ([x,y],[row,col]),Float)(∗\label{f2}∗)
Ix.defn = [ Case(c, Stencil(I(x,y), 1.0/12,(∗\label{d2}∗)
                    [[-1, 0, 1],
                     [-2, 0, 2],
                     [-1, 0, 1]]) ]

Ixx = Function(varDom = ([x,y],[row,col]),Float)(∗\label{f3}∗)
Ixx.defn = [ Case(c, Ix(x,y) ∗ Ix(x,y)) ](∗\label{d3}∗)

Iyy = Function(varDom = ([x,y],[row,col]),Float)(∗\label{f4}∗)
Iyy.defn = [ Case(c, Iy(x,y) ∗ Iy(x,y)) ](∗\label{d4}∗)

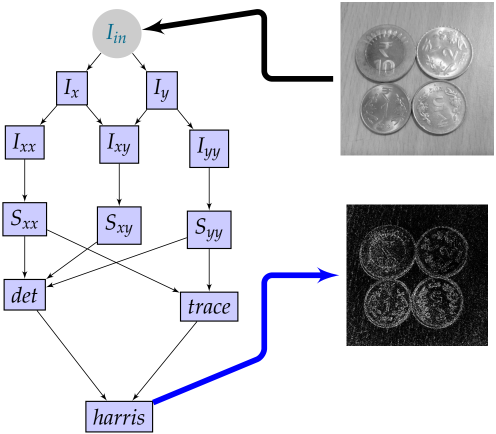
Ixy = Function(varDom = ([x,y],[row,col]),Float)(∗\label{f5}∗)
Ixy.defn = [ Case(c, Ix(x,y) ∗ Iy(x,y)) ](∗\label{d5}∗)

Sxx = Function(varDom = ([x,y],[row,col]),Float)(∗\label{f6}∗)
Syy = Function(varDom = ([x,y],[row,col]),Float)(∗\label{f7}∗)
Sxy = Function(varDom = ([x,y],[row,col]),Float)(∗\label{f8}∗)
for pair in [(Sxx, Ixx), (Syy, Iyy), (Sxy, Ixy)]:(∗\label{meta}∗)
    pair[0].defn = [ Case(cb, Stencil(pair[1], 1,(∗\label{d6}∗)
                              [[1, 1, 1],
                               [1, 1, 1],
                               [1, 1, 1]]) ]

det = Function(varDom = ([x,y],[row,col]),Float)(∗\label{f9}∗)
d = Sxx(x,y) ∗ Syy(x,y) - Sxy(x,y) ∗ Sxy(x,y)
det.defn = [ Case(cb, d) ] (∗\label{d7}∗)

trace = Function(varDom = ([x,y],[row,col]),Float)(∗\label{f10}∗)
trace.defn = [ Case(cb, Sxx(x,y) + Syy(x,y)) ](∗\label{d8}∗)

harris = Function(varDom = ([x,y],[row,col]),Float)(∗\label{f11}∗)
coarsity = det(x,y) - .04 ∗ trace(x,y) ∗ trace(x,y)(∗\label{d9}∗)
harris.defn = [ Case(cb, coarsity) ]
```



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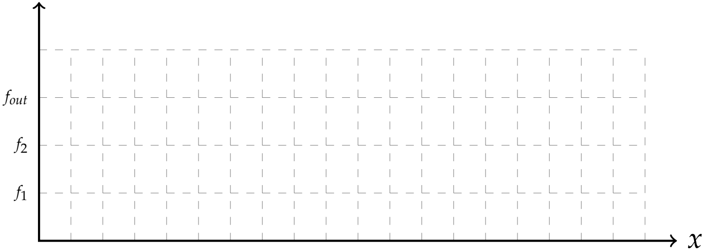
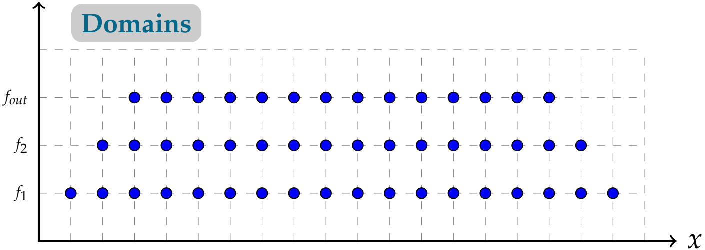
Polyhedral Representation




```
x = Variable()
f_in = Image(Float, [18])
f_1 = Function(varDom = ([x], [Interval(0, 17, 1)]), Float)
f_1.defn = [ f_in(x) + 1 ]
f_2 = Function(varDom = ([x], [Interval(1, 16, 1)]), Float)
f_2.defn = [ f_1(x-1) + f_1(x+1) ]
f_out = Function(varDom = ([x], [Interval(2, 15, 1)]), Float)
f_out.defn = [ f_2(x-1) + f_2(x+1) ]
```

Polyhedral Representation

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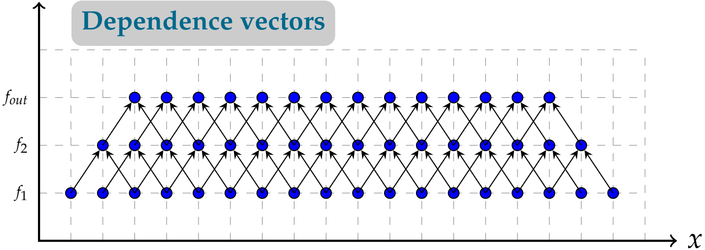
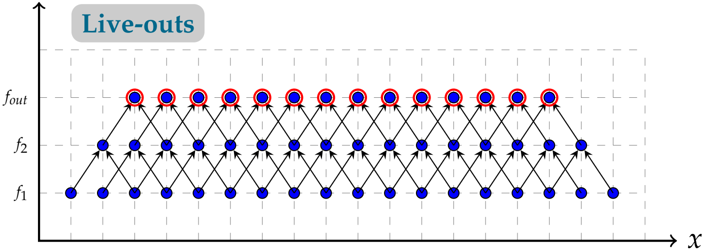


Function	Dependence Vectors
$f_{out}(x) = f_2(x-1) \cdot f_2(x+1)$	$(1,1), (1,-1)$
$f_2(x) = f_1(x-1) + f_1(x+1)$	$(1,1), (1,-1)$
$f_1(x) = f_{in}(x)$	

Polyhedral Representation



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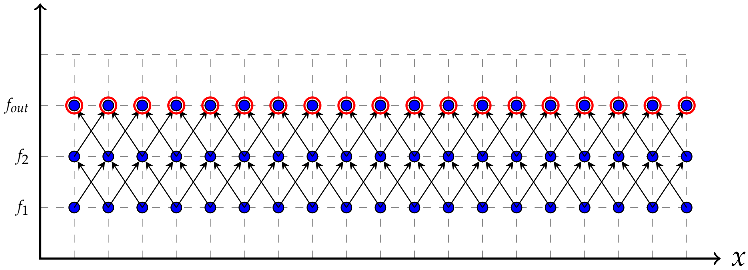
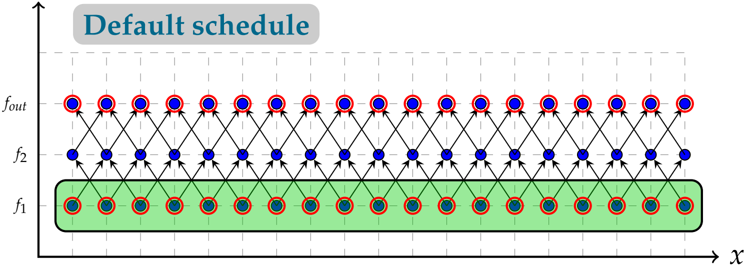
Scheduling Criteria



Parallelism **Locality** **Storage**

Scheduling Criteria

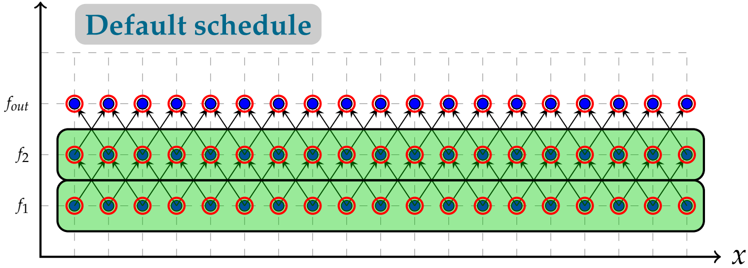


Parallelism

Locality

Storage

Scheduling Criteria



Parallelism

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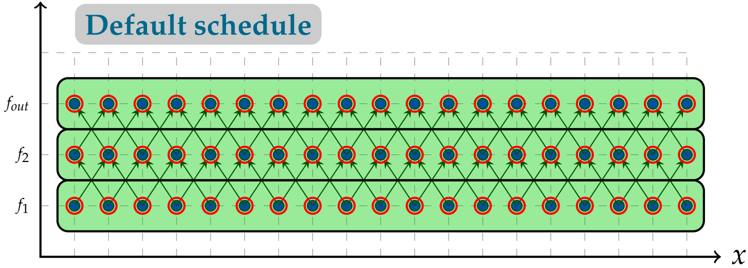

Parallelism

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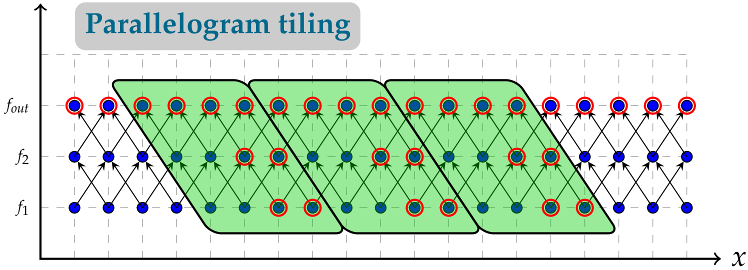

Parallelism

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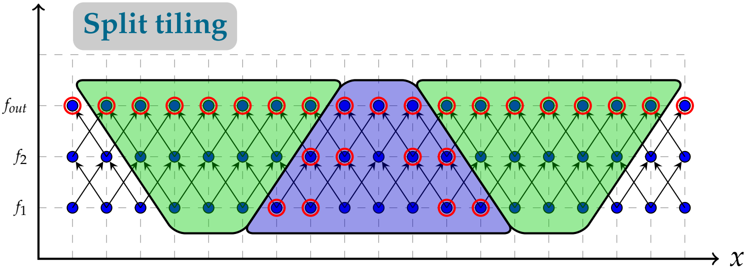

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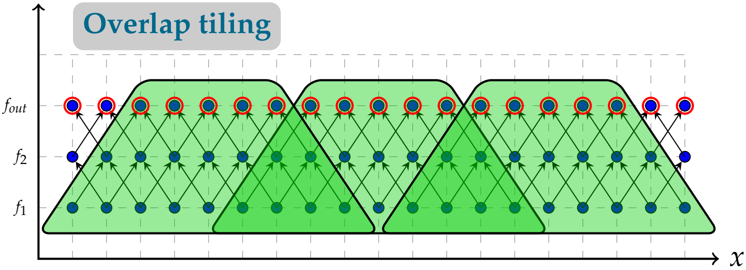


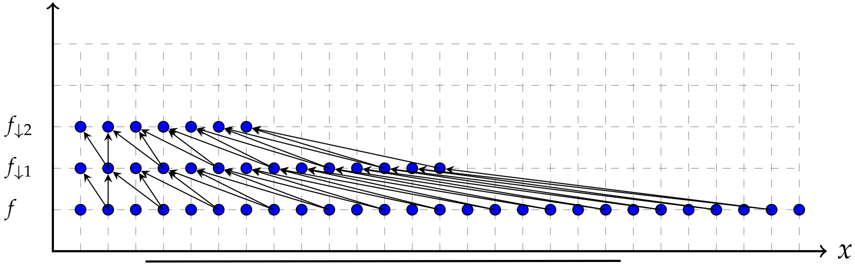
Parallelism

Locality

Storage

Re-computation

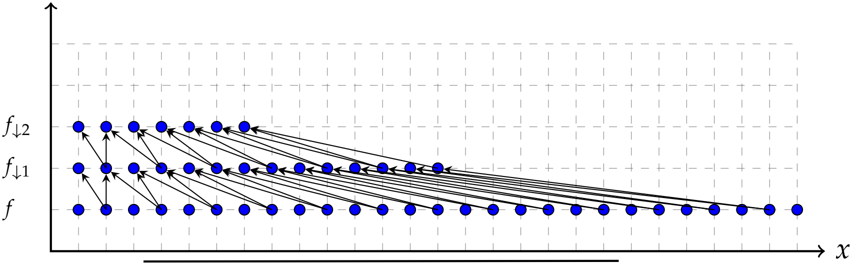
Overlapped Tiling for Heterogeneous Functions



Function	Schedule
$f_{\downarrow 2}(x) = f_{\downarrow 1}(2x-1) \cdot f_{\downarrow 1}(2x+1)$	$(x) \rightarrow (2, x)$
$f_{\downarrow 1}(x) = f(2x-1) \cdot f(2x+1) \cdot f(2x)$	$(x) \rightarrow (1, x)$
$f(x) = f_{in}(x)$	$(x) \rightarrow (0, x)$

- **Prior approaches for overlapped tiling only consider homogeneous time-iterated stencils**

Overlapped Tiling for Heterogeneous Functions



Function	Schedule
$f_{\downarrow 2}(x) = f_{\downarrow 1}(2x-1) \cdot f_{\downarrow 1}(2x+1)$	$(x) \rightarrow (2, x)$
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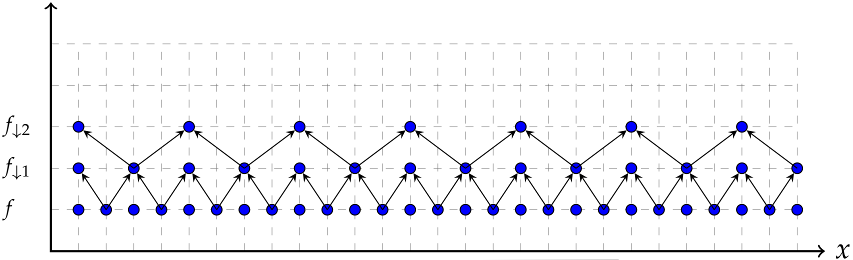
- **Cannot have a fixed tile shape when dependence vectors are non-constant**

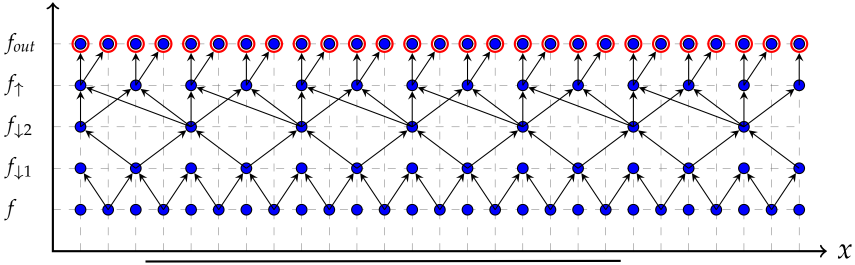
Overlapped Tiling for Heterogeneous Functions



Function	**Schedule**
$f_{\downarrow 2}(x) = f_{\downarrow 1}(2x-1) \cdot f_{\downarrow 1}(2x+1)$	$(x) \rightarrow (2, 4x)$
$f_{\downarrow 1}(x) = f(2x-1) \cdot f(2x+1) \cdot f(2x)$	$(x) \rightarrow (1, 2x)$
$f(x) = f_{in}(x)$	$(x) \rightarrow (0, x)$

- **Scaling and aligning the schedules**

Overlapped Tiling for Heterogeneous Functions



Function	Schedule
$f_{out}(x) = f_{\uparrow}(x/2)$	$(x) \rightarrow (4, x)$
$f_{\uparrow}(x) = f_{\downarrow 2}(x/2) \cdot f_{\downarrow 2}(x/2+1)$	$(x) \rightarrow (3, 2x)$
$f_{\downarrow 2}(x) = f_{\downarrow 1}(2x-1) \cdot f_{\downarrow 1}(2x+1)$	$(x) \rightarrow (2, 4x)$
$f_{\downarrow 1}(x) = f(2x-1) \cdot f(2x+1) \cdot f(2x)$	$(x) \rightarrow (1, 2x)$
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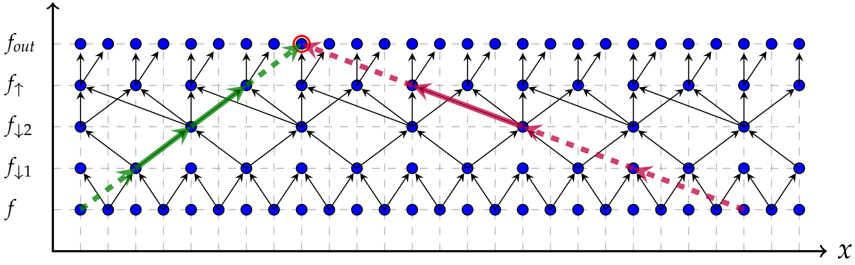
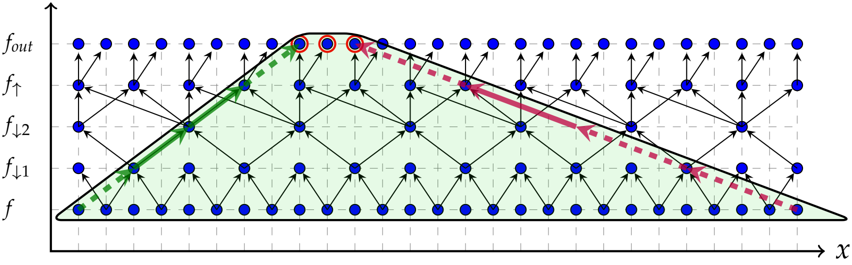


- **Determining tile shape**
  Conservative vs precise bounding faces

Overlapped Tiling for Heterogeneous Functions



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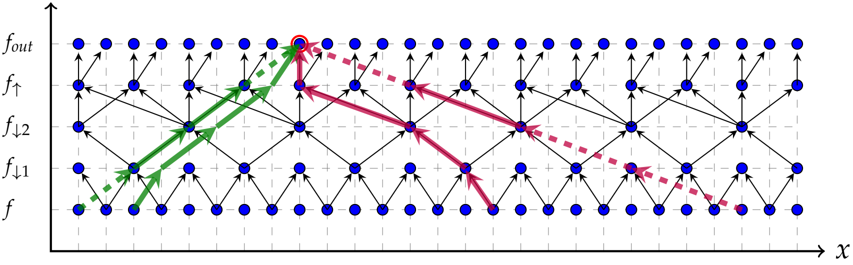

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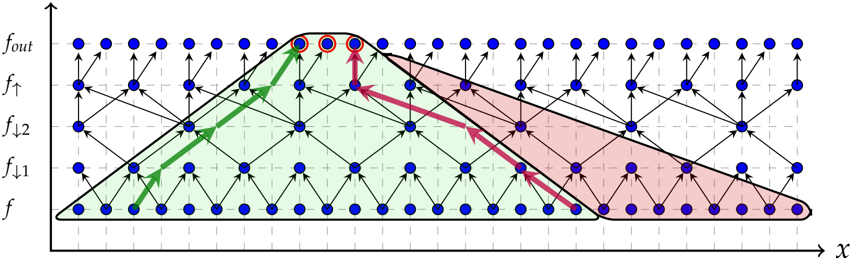
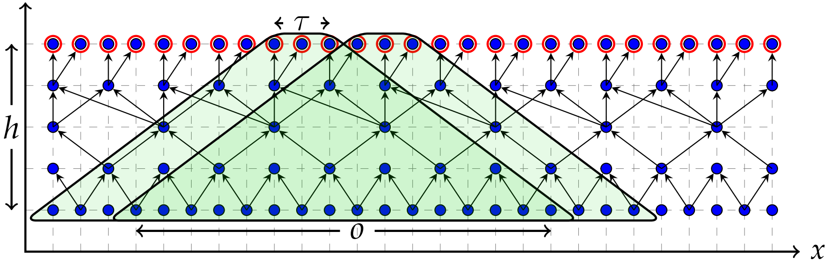


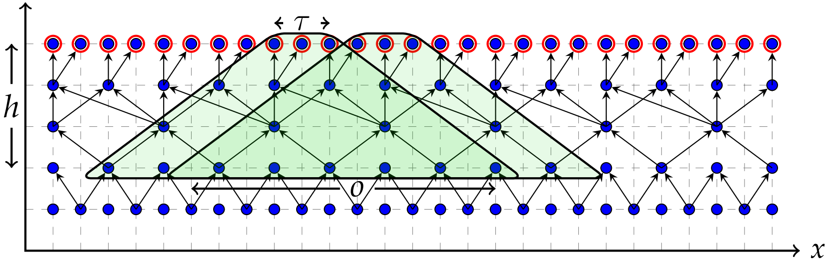
- **Significant reduction in redundant computation**

Overlapped Tiling for Heterogeneous Functions



- **Tile size** τ**, Overlap** O**, Height** h
  Trade-off between fusion height and overlap

Overlapped Tiling for Heterogeneous Functions



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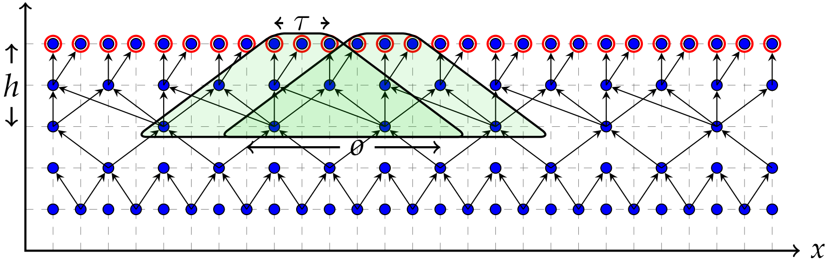

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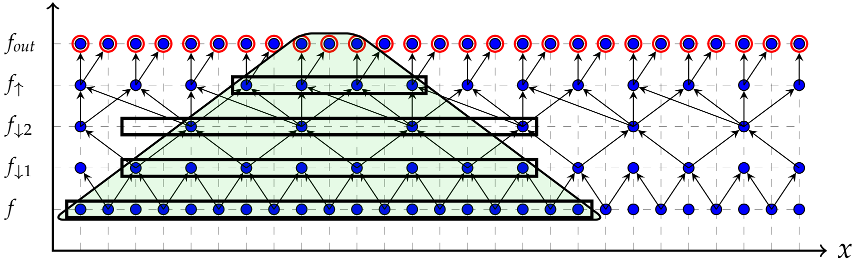
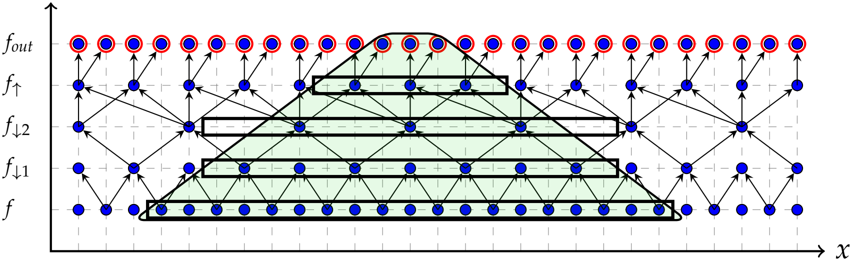


Scratch pads

- Reduction in intermediate storage
- Better locality and reuse
- Privatized for each thread

Overlapped Tiling for Heterogeneous Functions



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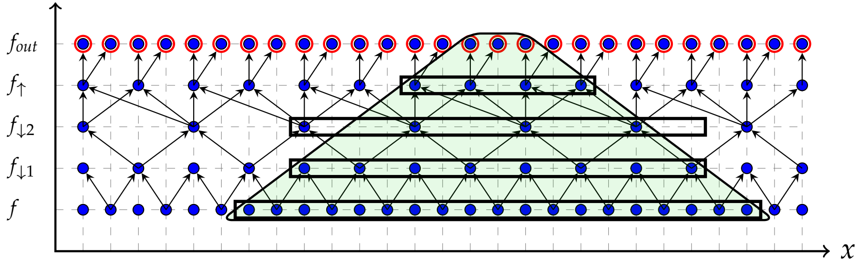

Scratch pads

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- Privatized for each thread

Benchmarks

Seven representative benchmarks of varying structure and complexity

Benchmark	Stages	Lines	Image size
Unsharp Mask	4	16	2048×2048×3
Bilateral Grid	7	43	2560×1536
Harris Corner	11	43	6400×6400
Camera Pipeline	32	86	2528×1920
Pyramid Blending	44	71	2048×2048×3
Multiscale Interpolate	49	41	2560×1536×3
Local Laplacian	99	107	2560×1536×3

Outline

1. Pluto/Pluto+
   - Affine Transformations
   - Tiling
2. Case Studies
   - Solving Partial Differential Equations
   - Lattice Boltzmann Method
   - Image Processing Pipelines
3. Conclusions

Conclusions

- Interesting to see how numerical techniques can be chosen and designed around modern parallel architectures and optimization infrastructure

Acknowledgements

- *Ravi Teja Mullapudi, Vinay Vasista, Irshad Pananilath, Aravind Acharya, Vinayaka Bandishti* (**students at IISc**)
- **Google Research**
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Thank You!